BIRS workshop 07w5063: **Explicit methods for rational points on curves**
Programme

Monday February 5, 2007

9:00 – 9:10 Introduction to BIRS

9:10 – 10:00 William McCallum – *Introduction to explicit Chabauty methods I*

Coffee

10:30 – 11:20 William McCallum – *Introduction to explicit Chabauty methods II*

11:30 **Lunch**

2:30 – 3:20 Richard Hain – *Higher Albanese Manifolds*

Tea

4:00 – 4:50 Kiran Kedlaya – *p-adic Hodge Theory*

At the request of the organizers, I will introduce/review some constructions from p-adic Hodge theory that intervene in the usual Chabauty method; these include the comparison isomorphism between the de Rham and etale cohomology groups of a curve, and the Bloch-Kato exponential map. I will focus on the case of good ordinary reduction, where these constructions can be made reasonably explicit. The goal is to analogize the explicit descriptions to the higher unipotent de Rham and etale fundamental groups, in a manner useful for doing nonabelian Chabauty; as time and my abilities permit, I will start doing this (again only in the good reduction case) using some work of Martin Olsson.

Tuesday February 6, 2007

9:10 – 10:00 Minhyong Kim, I

Coffee

10:30 – 11:20 Minhyong Kim, II

11:30 **Lunch**

2:30 – 3:20 Edward Schaefer – *Bounding the Mordell-Weil rank of the Jacobian of a curve*

We use a Chabauty computation to determine the set of rational points on a curve of higher genus. The input for a Chabauty computation includes the Mordell-Weil rank of the associated Jacobian. Traditionally we bound, and hope to determine, the Mordell-Weil rank using a Selmer group. In this talk, we will survey the methods for computing a Selmer group of a Jacobian using functions on the curve. We will review both major methods. The first is quite general, but is inefficient for cyclic covers of the projective line (like hyperelliptic curves). The second method addresses such covers.

Tea

4:00 – 4:50 Michael Stoll – *Local-global obstructions, coverings, and Mordell-Weil sieving*

We will discuss how one can obtain information on rational points by combining coverings with local information. We will focus on the case of abelian coverings and explain the relationship with the Brauer-Manin obstruction. If explicit generators of the Mordell-Weil group are known, this can be implemented efficiently, leading to a procedure known as the Mordell-Weil sieve. We will formulate a conjecture that, if valid for a given curve, implies that we can effectively decide whether a given coset of N times the Mordell-Weil group meets the image of the curve or not. If we know that each such coset contains at most one point coming from the curve, this means that we can determine the set of rational points on the curve.

5:00 – 5:50 Jordan Ellenberg – *Obstructions to rational points on curves coming from the nilpotent geometric fundamental group*

Wednesday February 7, 2007

NOTE: This schedule can be moved forward to optimise daylight time for skiers

9:10 – 10:00 Minhyong Kim, III

Coffee

10:30 – 11:20 Samir Siksek – *Chabauty for Symmetric Powers of Curves*

Let C be a curve of genus $g \geq 3$ and let $C^{(d)}$ denote its d-th symmetric power. We explain an adaptation of Chabauty which allows us in many cases to compute $C^{(d)}(\mathbf{Q})$ provided the rank of the Mordell-Weil group is at most $g - d$. Cases for which our method should work include:

(i) $d < \gamma$ where γ is the gonality of C and the jacobian is simple (here $C^{(d)}(\mathbf{Q})$ is finite).

(ii) C is hyperelliptic and $d = 2$ (here $C^{(d)}(\mathbf{Q})$ is infinite).

(iii) C is bielliptic and $d = 2$ (here $C^{(d)}(\mathbf{Q})$ can be infinite). Our adaptation of Chabauty differs from the classical Chabauty in that we combine Chabauty type information given by several primes.

Example. Let C be the genus 3 hyperelliptic curve

$$C : y^2 = x(x^2 + 2)(x^2 + 43)(x^2 + 8x - 6) \tag{1}$$

with Jacobian having rank 1. Let $\pi : C \to P^1$ be the x-coordinate map. We show that $C^{(2)}(\mathbf{Q})$ consists of $\pi^{-1}P^1(\mathbf{Q})$ plus 10 other points which we write down explicitly. Here we needed to combine the Chabauty information at primes $p = 5$, 7, 13. It is noteworthy that $C^{(2)}$ in this example is a surface of general type.

11:30 **Lunch**

Skiing Afternoon

Thursday February 8, 2007

9:10 – 10:00 Tim Dokchitser – *Analytic ranks of Jacobians of curves*

Coffee

10:30 – 11:20 Ronald van Luijk – *Cubic points on cubic curves and the Brauer-Manin obstruction for K3 surfaces*

It is well-known that not all varieties over $\mathbb{Q}$ satisfy the Hasse principle. The famous Selmer curve given by $3x^3 + 4y^3 + 5z^3 = 0$ in $\mathbb{P}^2$, for instance, indeed has points over every completion of $\mathbb{Q}$, but no points over $\mathbb{Q}$ itself. Though it is trivial to find points over some cubic field, it is a priori not obvious whether there are points over a cubic field that is galois. We will see that such points do exist. K3 surfaces do not satisfy the Hasse principle either, which in some cases can be explained by the so called Brauer-Manin obstruction. It is not known whether this obstruction is the only obstruction to the existence of rational points on K3 surfaces. We relate the two problems by sketching a proof of the following fact. If there exists a smooth curve over $\mathbb{Q}$ given by $ax^3 + by^3 + cz^3 = 0$ that is locally solvable everywhere, that has no points over any cubic galois extension of $\mathbb{Q}$, and whose Jacobian has trivial Mordell-Weil group, then the algebraic part of the Brauer-Manin obstruction is not the only one for K3 surfaces. No knowledge about K3 surfaces or Brauer-Manin obstructions will be assumed as known.

11:30 **Lunch**

2:30 – 3:20 Catherine O'Neil – *Trilinear forms and elliptic curves*

We explain a correspondence between triliear forms and triples of genus one curves with a fixed Jacobian and some added structure. We generalize the addition law on elliptic curves to addition on certain "cubes" of numbers. We explain how this works for arbitrary rings, and we give a natural construction of points on elliptic curves to other points on other elliptic curves which generalizes a known construction from class field theory.

Tea

4:00 – 4:50 William Stein – *Explicilty computing information about Shafarevich-Tate groups of elliptic curves using L-functions, Euler Systems, and Iwasawa theory*

I will discuss theoretical and computational results toward the following problem: given a specific elliptic curve over $\mathbb{Q}$, compute the exact order and structure of its Shafarevich-Tate group in practice. I view this problem as a motivating question for organizing both theoretical and algorithmic investigations into the arithmetic of elliptic curves and the Birch and Swinnerton-Dyer conjecture.

5:00 – 5:30 Iftikhar Burhanuddin – *Brauer-Siegel Analogue for Elliptic Curves over the Rationals*

The height of a rational point on an elliptic curve measures the size of the point. The enormous gap between the lower and upper bound (Lang's conjectures) of the height of such a point, prompted the comparison of the elliptic curve scenario with that of the multiplicative group, the Brauer-Siegel theorem. In this talk, a conjectural Brauer-Siegel theorem for elliptic curves over the rationals will be discussed and interesting questions which arise in this context motivated by computation will be presented.

Friday February 9, 2007

9:10 – 10:00 Florian Hess – *Explicit generating sets of Jacobians of curves over finite fields, using some class field theory*

Coffee

10:30 – 11:20 Michael Stoll – *Rational points on small curves of genus 2 - an experiment*

We considered all genus 2 curves $y^2 = f(x)$ where f has integral coefficients of absolute value at most 3; there are about 200,000 isomorphism classes of such curves. Using various methods (point search, local solubility, 2-descent, Mordell-Weil sieve), we attempted to decide for each curve whether it possesses rational points. In all but 42 cases, we were successful; in the remaining cases, our result is conditional on the Birch and Swinnerton-Dyer conjecture. In the talk, we will explain the methods we used and the improvements we came up with, and dicuss the results.

11:30 **Lunch**

Workshop ends with lunch