Invited lectures

Vadim Kaloshin

Pavao Mardesic : Infinitesimal and tangential 16-th Hilbert problem

Abstract: In this lecture we present some principal results on the infinitesimal and tangential 16-th Hilbert problem. We explain some techniques and difficulties and give some perspective to the numerous open problems.

Abdelraouf Mourtada: "Hilbert's 16th problem for hyperbolic polycycles and extension of Khovanskii-Varchenko theorem to algebraic polycycles"

We show that there is no accumulation of limit cycles on a hyperbolic polycycle, in compact families of analytic vector fields on the sphere S^2. The proof includes the case where the polycycle is an accumulation of cycles. Then, we use the ideas of this proof, to extend the result of Khovanski-Varchenko about Abelian integrals, to the neighbourhood of hyperbolic polycycles. And this gives rise to the following general result: let H be a Morse polynomial of degree d+1 which is generic at infinity (but maybe with multiple critical values). Then there exist a number N(d) (depending only on d), such that every perturbation of dH (of degree d and with non vanishing Abelian integrals) has at most N(d) limit cycles on the real plane.

Robert Roussarie: "Slow-fast systems and Sixteenth Hilbert's Problem"

Abstract

The existential version of the 16th Hilbert's problem reduces to prove that any limit periodic set in a compact analytic family of vector fields on a 2-sphere has a finite cyclicity. The more complicated limit periodic sets are the degenerate graphics which contain non isolated singular points. In a neighborhood of such a degenerate graphics the family is equivalent to a slow fast system. To simplify, I shall just consider slow fast Lienard systems. I shall recall some basic definitions and comment recent results, in particular about the bifurcation of canard cycles.

Jean-Christophe Yoccoz: « Siegel disks and Julia sets of quadratic polynomials, according to X.Buff and A.Chéritat ».

Abstract: We will report on the recent work of X.Buff and A.Chéritat on quadratic polynomials with an indifferent fixed point. They obtain deep results on the geometry and size of Siegel disks which allow them in particular to find parameters for which the corresponding Julia set has positive Lebesgue measure.

New lectures received on March 8: Glutsyuk and Gasull and Teyssier

Emmanuel Paul : "Galoisian reducibility for a germ of quasi-homogeneous foliation"

Christiane Rousseau : The space of modules of unfoldings of germs of generic diffeomorphisms with a parabolic point

Armengol Gasull: Some results on periodic orbits for Abel-type equations

Consider the
Abel-type differential equations
$$
\frac{dx}{dt}=a_n(t)x^n+a_{n-1}(t)x^{n-1}+\cdots+ a_1(t) x+a_0(t),
$$
where $x\in \bf{R}$, $t\in[0,1]$ and $a_0,a_1,\dots,a_n : \bf{R}\to\bf{R}$, are smooth 1-periodic functions. These equations can be thought in the cylinder and a classical problem is to study how many limit cycles they can have in terms of n and some hypotheses of the function $a_j(t).$ The aim of this talk is to give a survey on some recent results obtained by the speaker and several collaborators about this question, see the references below.

Special attention is devoted to the following two subclasses of differential equations:
$$
\frac{dx}{dt}= a_n(t)x^n +a_2(t)x^2+a_1(t)x+a_0(t),$$ with $n>2$ and
$$
\frac{dx}{dt}=a_n(t)x^n+a_{m}(t)x^{m}+ a_1(t) x,
$$
where $n>m>1.$ Moreover we consider with more detail the usual Abel equation ({\it i.e.} the case $n=3$ in the above equations) when the functions $a_j(t)$ are trigonometrical polynomials.

Freddy Dumortier : The period function of classical Liénard equations.

We want to present recent work obtained in collaboration with Peter De Maesschalck. The talk deals with the number of critical points that the period function of a center of a classical Liénard equation can have. We show that proving the existence of a finite upperbound on the number of critical periods, only depending on the degree of the friction term, can be reduced to the study of slow-fast Liénard equations close to their limiting layer equations. We provide a sharp upperbound for the number of critical periods near a central system. We show the occurrence of slow-fast Liénard systems, having the same number of critical periods, elucidating a qualitative process behind the occurrence of critical periods. We present a conjecture on the maximal number of critical periods.

Rafel Prohens Sastre: On the number of limit cycles of some systems on the cylinder

(jointly with: M.J.Álvarez and A. Gasull) When one consider systems of ordinary differential equations on a cylinder, that is systems in (ρ,θ)-polar coordinates where all the functions defining the system are 2π-periodic in θ, then two types of periodic orbits can be considered. The ones that can be deformed continuously to a point, which we call {\it contractible} periodic orbits, and the ones that can not, which we call {\it non-contractible} periodic orbits. As usual, a periodic orbit isolated in the set of all the periodic orbits is called {\it limit cycle}.

In this work we will focus our attention on the non-contractible limit cycles and what we present are results that allow to control the number of such limit cycles.

These results apply, not only to systems on the cylinder, but even to systems coming from planar differential equations. As application, we present two criteria for proving uniqueness of limit cycles surrounding the origin in cubic systems with a symmetry of order 4.

Patrick Bonckaert : 'Invariant manifolds close to linear non-hyperbolic singularities', 25 minutes

Magdalena Caubergh : 'Large Amplitude Limit Cycles for Liénard systems'

Jaume Llibre : On the limit cycles of the Liénard differential systems

ABSTRACT: One of the main interesting problems in the qualitative theory of planar differential equations is the classical problem of studying their limit cycles. When the differential equations are polynomial this is the well known 16th Hilbert's problem.

A particular case of the 16th Hilbert's problem is the study of the limit cycles of the Lienard systems of the form x' = y - F(x), y'= -x, where F(x) is a polynomial. For these systems there exists the conjecture of Lins, de Melo and Pugh about their number of limit cycles, revisited by Smale later on. We will talk about this problem, and present old and new results on it.

Rodica Costin
Title: Nonlinear perturbations of Fuchsian systems: linearization criteria and classification

Abstract: Linearization of a differential system (the fact that a given equation is analytically equivalent to a linear equation, in a given region) is an important property of an equation, and is connected to integrability.

It is shown that differential systems which are nonlinear perturbations of Fuchsian systems are not analytically linearizable, generically. Obstructions to linearization are found, and proved that they imply nonintegrability, in the sense of absence of single-valued first integrals.

It is also shown that there exists a unique autonomous correction of any vector field so that the "corrected system" is linearizable.

Orthogonal polynomials appear naturally in these problems, and relate to integrability as well.

Reinhard Schäfke: Quasi-analytic solutions of analytic ordinary differential equations and O-minimal Structures

Loïc Teyssier: "Confluence of singular points in a family of holomorphic vector fields"

ABSTRACT:
We consider analytic families $(Z_\varepsilon)_\varepsilon$ of holomorphic vector fields in the complex plane, unfolding a saddle-node singularity. This happens when "simple" singular points collide and merge into a multiple singular point. We characterize the equivalence classes of such families under (local) analytic changes of coordinates and parameter. We do so by conjugating each family to a model one and by comparing the normalizing maps. This procedure yields functional invariants which unfold the invariants of Martinet and Ramis for the corresponding saddle-node. The invariants are obtained by identifying the obstructions to solving two cohomological equations of the form $Z_{\varepsilon}(F)=G$. One equation governs the orbital part of the modulus (that is, the modulus for the family of underlynig foliations) while the other one provides the time part of the modulus. These obstructions correspond to periods of G, which are expressed as integrals along paths tangent to the vector field and connecting singular points of Z_ε. On adapted spiraling sectors where such paths can't turn up we obtain analytic conjugacies to the model family. This construction generalizes and gives a geometric proof to the theorem of Hukuhara, Kimura and Matuda addressing saddle-nodes foliations.

Tanya Firsova: "Topology of analytic foliations on Stein manifolds".

Dmitry Novikov: "Extension of the Varchenko-Khovanskii theorem to the integrable case"

Javier Ribon
Title: Analytic classification of unfoldings of resonant diffeomorphisms

Abstract:

We provide a complete system of analytic invariants for unfoldings of non-linearizable resonant complex analytic diffeomorphisms; it is a generalization of the Mardesic-Roussarie-Rousseau system for the generic case. Moreover, we give an interpretation of the invariants leading us to obtain a geometric version of the theorem of analytic classification.

In order to fulfill this goal we develop an extension of the Fatou coordinates with controlled asymptotic behavior in the neighborhood of the fixed points. The classical constructions are based on finding regions where the dynamics of the unfolding is topologically stable. We introduce a concept of infinitesimal stability leading to Fatou coordinates reflecting more faithfully the analytic nature of the unfolding. These improvements allow us to control the domain of definition of a conjugating mapping and its power series expansion.

Huaiping Zhu
Title: Bifurcation of limit cycles from a Nilpotent Center in a Near-Hamiltonian System

Caroline Lambert: Confluence of the hypergeometric equation and Riccati equation

Joan C. Artés:
Quadratic vectors fields of codimension

Alexey Glutsyuk: On density of horospheres in dynamical laminations

In 1985 D.Sullivan had introduced a dictionary between two domains of complex dynamics: iterations of rational functions and Kleinian groups (both acting on the Riemann sphere). This dictionary motivated many remarkable results in both domains, starting from the famous Sullivan's no wandering domain theorem in the theory of iterations of rational functions.

One of the principal objects used in the study of Kleinian groups is the hyperbolic 3- manifold associated to a Kleinian group, which is the quotient of its lifted action to the hyperbolic 3- space.
M.Lyubich and Y.Minsky have suggested to extend Sullivan's dictionary by providing an analogous construction for iterations of rational functions: hyperbolic laminations. Appropriate surgery on the backward orbit space yields an abstract topological space foliated by hyperbolic 3- manifolds (with singularities). The nonbijective action of the rational function lifts up to a bijective action on the latter space by isometries of leaves. The quotient of the latter action is a nice space (called the quotient hyperbolic lamination). It is foliated by hyperbolic 3- manifolds (may be with singularities) with marked point "infinity" on the boundaries of their covering hyperbolic spaces.
The horospheres passing through infinity (i.e., the horizontal planes of the hyperbolic 3- space in the half-space model) induce a foliation by surfaces in the quotient lamination space (called the horospheric lamination).

Recent studying the hyperbolic 3- manifolds associated to Kleinian groups resulted in solutions of all big problems in the theory.
There is a hope that studying the hyperbolic laminations associated to rational functions would imply important dynamical corollaries.

One of the main results of the talk says that the horospheric lamination is topologically transitive, provided that the rational function under consideration does not belong to an explicit list of exceptions (for which this is not true).