Banff International Research Station

for Mathematical Innovation and Discovery

Moduli Spaces and Combinatorics
July 22nd – July 27th, 2006

MEALS

Breakfast (Continental): 7:00–9:00 am, 2nd floor lounge, Corbett Hall, Sunday–Thursday
*Lunch (Buffet): 11:30 am–1:30 pm, Donald Cameron Hall, Sunday–Thursday
*Dinner (Buffet): 5:30–7:30 pm, Donald Cameron Hall, Saturday–Wednesday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each lunch and dinner.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by bridge on 2nd floor of Corbett Hall). Hours: 6 am–12 midnight. LCD projector, overhead projectors and blackboards are available for presentations. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

SCHEDULE

Saturday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Donald Cameron Hall
20:00	Informal gathering in 2nd floor lounge, Corbett Hall
	Beverages and small assortment of snacks available on a cash honour-system.

Sunday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
9:00–10:00	Okounkov I
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	Maulik
11:30–13:30	Lunch
13:30–14:30	Borodin
14:30–15:00	Coffee Break, 2nd floor lounge, Corbett Hall.
15:00–16:00	Pandharipande
16:30–17:30	Knutson
17:30–19:30	Dinner

Monday

7:00–9:00	Breakfast
9:00–10:00	Okounkov II
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	Szendroi
11:30–13:30	Lunch
13:30–14:30	Mikhalkin
14:30–15:00	Coffee Break, 2nd floor lounge, Corbett Hall.
15:00–16:00	Proudfoot
16:00–16:15	Group Photo; meet on the front steps of Corbett Hall.
17:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–10:00	Okounkov III
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	Vakil
11:30–13:30	Lunch
	Free afternoon!
17:30–19:30	Dinner

Wednesday

7:00–9:00	Breakfast
9:00–10:00	Jackson
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	Goulden
11:30–13:30	Lunch
13:30–14:30	Craw
14:30–15:00	Coffee Break, 2nd floor lounge, Corbett Hall.
15:00–16:00	Maclagan
16:30–17:30	Faber
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00–10:00	Farkas
10:00–10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30	Informal discussions
11:30–13:30	Lunch

Checkout by 12 noon.

** 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Thursday, although participants are still required to checkout of the guest rooms by 12 noon. **

Moduli Spaces and Combinatorics
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ABSTRACTS

Speaker: **Grisha Mikhalkin** (University of Toronto)
Title: *Curves in P^n: inductive enumeration by dimension.*
Abstract:

Speaker: **Nick Proudfoot** (University of Texas at Austin)
Title: *Moduli spaces for Bondal quivers*
Abstract: Much of the data of an algebraic variety is encoded by its (derived) category of coherent sheaves, and I will explain how it is sometimes possible to 'present' this category as the (derived) category of representations of a quiver. In such a situation, it is natural to consider the moduli space of stable representations of the quiver, which depends sensitively on your notion of stability. I will discuss the relationship between this moduli space and the original variety.

Speaker: **Carel Faber** (Johns Hopkins University and KTH)
Title: *Combinatorial aspects of the tautological ring*
Abstract: There exists a conjectural description of the tautological ring of the moduli space of curves, which has been verified for all $g < 24$. Only the conjectured perfect pairing is not known yet for all g. For any potentially generating set of relations, it is a combinatorial problem to prove generation and the perfect pairing simultaneously. I will discuss several sets of relations. Independently, one can study the Gorenstein quotient of the tautological ring; Zagier and I have obtained some results here. If time permits, I will discuss some analogous problems for related moduli spaces of curves.

Speaker: **Alastair Craw** (Stony Brook University
Title: *Projective toric varieties are fine moduli spaces*
Abstract: Given a collection $\mathcal{L}$ of line bundles on a projective toric variety X, I will introduce the corresponding "multilinear series" $|\mathcal{L}|$ as a moduli space of representations of a quiver. Under mild hypotheses, one can embed X into this space, generalizing the morphism determined by a single very ample line bundle on X.

The combinatorics becomes interesting when you compare the image of X in $|\mathcal{L}|$ with the subscheme cut out by certain natural relations in the quiver; under stronger hypotheses, X coincides with (the coherent component) of this subscheme, and hence X itself is a fine moduli space of representations of the quiver with relations. This programme leads to new examples of full strong exceptional collections of line bundles on certain Fano toric varieties, and hence to new examples where the derived category of a toric variety is well understood. This is joint work with Gregory G. Smith (Queens Univ.).

Speaker: **Allen Knutson** (UCSD)
Title: *Moduli spaces of branchvarieties, and combinatorics of Littelmann paths*
Abstract: The moduli space of subschemes of projective space is complete and separated, but not of finite type until one fixes a Hilbert polynomial. The resulting "Hilbert scheme" is connected, i.e. has no further local invariants.

I'll present an alternative to subschemes, "branchvarieties", which are reduced schemes with a finite map to projective space. (Both concepts generalize subvarieties – one gives up variety, the other gives up sub.) These form a complete, separated moduli stack, but to obtain one of finite type one must fix more invariants, such as a labeled rooted forest. This work is joint with Valery Alexeev.

If time permits, I'll describe how Chirivi's geometric interpretation of the Littelmann-Lakshmibai-Seshadri weight multiplicity formula is a case of a limit in the moduli stack of branchvarieties.

Speaker: **Gavril Farkas** (University of Texas at Austin)
Title: *Koszul divisors on moduli spaces*
Abstract: We present a very general method of constructing "exceptional" divisors on a large class of moduli spaces using the syzygies of the parametrised objects. The method can be applied to a large class of spaces (moduli spaces of curves (with or without level structure), moduli of surfaces etc) and among the applications we mention:

- a proof that the moduli space of curves of genus 22 is of general type.
- a proof that the moduli space of Prym varieties of dimension greater than 12 is of general type.
- an infinite string of counter-examples to the Harris-Morrison Slope Conjecture on effective divisors on moduli spaces of curves.
- a simpler (and unified) way of rederiving all the divisor class calculations on the moduli space of curves due to Harris, Mumford and Eisenbud.

Speaker: **Alexei Borodin** (Cal Tech)
Title: *Periodic Schur process*
Abstract:

Speaker: **Balazs Szendroi** (Oxford University)
Title: *D-branes on ADE fibred surfaces*
Abstract: I show how to classify certain holomorphic D-branes, closely related to framed torsion-free sheaves, on threefolds fibered in ADE surfaces over curves. For geometries over the affine line, this recovers a quiver problem studied earlier by Cachazo-Katz-Vafa in the context of quiver gauge theory.

Speaker: **Rahul Pandharipande** (Princeton University)
Title: *Gromov-Witten theory of irrational surfaces*
Abstract:

Speaker: **Andrei Okounkov** (Princeton University)
Title: *Three lectures on Hilbert schemes and symmetric functions: 1) Introduction to Hilbert schemes, 2) The topological vertex, 3) The equivariant vertex.*
Abstract: My goal will be to give an elementary introduction to the various combinatorial structures related to Hilbert schemes of points and curves. The first lecture will be a leisurely survey of the geometry of Hilbert schemes of points in the plane and its relations to various natural bases in the symmetric functions, especially Jack polynomials. In the second lecture, we will begin to look at points and curves in 3-folds, starting with a special case of the topological vertex and its evaluation in terms of Schur functions. The full equivariant vertex and some of its properties will be the subject of the third lecture, if time permits.

Speaker: **Diane Maclagan** (Rutgers)
Title: *Moduli of representations of the McKay quiver for Abelian groups*
Abstract: When G is a finite subgroup of $SL(3)$ the moduli space M_θ of representations of the McKay quiver is a crepant resolution of the quotient singularity C^3/G. I will describe joint work with Alastair Craw and Rekha Thomas giving an explicit description of the component of M_θ that is birational to C^n/G for Abelian G in $GL(n,\mathbb{C})$ for arbitrary n as a (not necessarily normal) toric variety defined by the data of the quiver. A special case of the moduli of McKay quiver representations is Nakamura's G-Hilbert scheme, and our explicit description allows us to construct pathological examples of these schemes

Speaker: **David Jackson** (Waterloo)
Title: *The combinatorics of localisation trees and Faber's intersection number conjecture*
Abstract: I consider two constructions discussed by Vakil, namely degeneration and localisation, but now in the context of enumerative combinatorics. Degeneration leads to a Join-cut partial differential equation for the Faber- Hurwitz series, while localisation leads to a sum over localisation trees. I warm up with the Join-cut equation for the genus 0 Hurwitz numbers and a proof of Hurwitz's 1891 result through a combinatorial analysis. (This work is joint with Ian Goulden and Ravi Vakil.)

Speaker: **Ian Goulden** (Waterloo)
Title: *Ring theoretic formal analysis and the proof of Faber's intersection number conjecture for a few parts.*
Abstract: I consider a fundamental equation discussed by Jackson, namely that the Faber-Hurwitz series is equal to a transformed localisation tree series. The development of the necessary ring theoretic formal series. The development of the necessary ring theoretic formal analysis is described and is used to obtain a proof of Faber's intersection number conjecture for up to three points and all genera. I warm up with a description of our recent proof the the λ_g-Conjecture and Kazarian and Lando's recent proof of Witten's Conjecture. (This is joint with David Jackson and Ravi Vakil.)

Speaker: **Davesh Maulik** (Princeton University)
Title: *Gromov-Witten theory of A_n resolutions*
Abstract:

Speaker: **Ravi Vakil** (Stanford University)
Title: *Combinatorialization of Faber's intersection number conjecture*
Abstract: The moduli space of curves is one of the most fundamental spaces in algebraic geometry. The last decade has seen dramatic progress in our understanding of this space, driven in no small part by ideas from other fields of mathematics (and thanks in no small part to other participants of this conference). Much of our understanding of this space centers around Faber's conjectures. Faber's intersection number conjecture gives a beautiful combinatorial description of the "top intersections" on this moduli space, and in this talk I will sketch how one can "combinatorialize" these intersections. (This conjecture is now a theorem, thanks to Getzler and Pandharipande, and powerful machinery of Givental, but our goal is to explain more directly why it is true.)

In the first half, I will sketch how questions about factorizations in the symmetric group can be reinterpreted geometrically ("Hurwitz theory"), and then approached by the geometric means of localization, and the hybrid means of degeneration/join-cut. In the second half, I will sketch how to apply similar techniques to cohomology classes, and hence how to combinatorialize "top intersections", prove geometric facts along the way, and explain why the outcome is prime for combinatorial analysis.