HYPERBOLIC SYSTEMS OF CONSERVATION LAWS AND RELATED PROBLEMS

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1 Overview of the Field

Systems of Conservation Laws result from the balance law of continuum physics and govern a broad spectrum of physical phenomena in compressible fluid dynamics, nonlinear materials science, particle physics, semiconductors, combustion, multi-phase flows, astrophysics, relativity, and other applied areas. Typical examples of "nonlinear conservation laws" are the Euler equations, MHD equations, Navier Stokes equations, Boltzmann equation, Einstein equation, and other important models arising in Elasticity, Fluid Dynamics, Combustion, Kinetic Theory, and Relativity.

The Euler equations for inviscid compressible fluid flow are a core system that is fundamental and important to many applications, yet the multidimensional theory is difficult and challenging.

In recent years, major progress has been made in both the theoretical and numerical aspects of the field. Motivated by recent results that are bound to revolutionize the field and by major open problems with great relevance to applications, we organized a 5-day workshop at Banff bringing together experts in the theoretical and numerical aspects of hyperbolic conservation laws and related models with focus on applications.

2 Recent Developments and Open Problems

The theme of the workshop covered several aspects of the theory of weak solutions for hyperbolic systems, the mathematical theory of transport equations that arise in the kinetic theory of gases, and the investigation of the multidimensional Euler, relativistic Euler, Euler-Poisson, and Navier-Stokes equations.

In the area of multidimensional hyperbolic systems, we discussed topics of research in three basic areas, all unified by their showed common dependence on mixed hyperbolic-elliptic equations and geometry:

- Unsteady transonic flow and shock reflection;
- Steady transonic flow and differential geometry;
- Mixed systems arising in blood flow modeling.

This research is related to problems in two or more space dimensions. We feel that the subject of conservation laws in one space dimension is relatively well developed. It is the multidimensional area, especially for the nonlinear conservation laws of mixed hyperbolic-elliptic type, where major effort is needed.

In the area of Navier-Stokes equations in several space dimensions, existing results and major open problems were discussed on questions of existence and regularity of solutions. Further presentations on models of compressible fluids were given with focus on applications.

3 Presentation Highlights

In this section we present a description of the keynote lectures of the meeting. The main themes of the *keynote* lectures are:

- *"Navier-Stokes equations for compressible fluids: Existence Results"* by David Hoff;
- *" Vanishing viscosity method for transonic flow"* by Marshall Slemrod;
- *"Existence and regularity of solutions to shock reflection problems"* by Mikhail Feldman;
- *"Thin film equation for the flow of a thin layer of fluid down an inclined plane – Overcompressible waves"* by Michael Shearer.

More details for the keynote lectures are the following:

3.1 A Survey of Existence Results for the Navier-Stokes Equations of Compressible Fluid Flow

We give a survey of results on the existence of solutions of the Navier-Stokes equations of multidimensional, compressible fluid flow. These equations model the conservation of mass and the balance of momentum and energy in terms of density, velocity, and temperature, which are the unknown functions of space and time. See Batchelor [1] for a derivation of these equations and for a discussion of the underlying physics.

We describe three categories of solutions. In the first group are the *small-smooth* solutions of Matsumura–Nishida in [12, 13]. These solutions are obtained by linearizing the equations, absorbing the linearization errors in Duhamel integrals, and iterating. By applying asymptotic decay rates associated to the linearized solution operator, one can then obtain convergence of the iterates in global time under the assumption that the initial data is small in a norm which dominates derivatives. The difficult part of the analysis is obtaining the decay rates, because there is no parabolic dissipation in the mass equation. This approach of Matsumura and Nishida has been optimized by Danchin [2, 3] for the constant coefficient case, who replaces the Sobolev space H^3 occurring in the Matsumura-Nishida theory with certain Besov spaces of continuous functions. These spaces have more appropriate scaling properties, and, remarkably, are nearly algebras. Small-smooth solutions by their very nature do not exhibit nonlinear effects, however, and tell us rather little about fluid flow.

At the opposite end of the spectrum are the very general weak solutions introduced by Lions, which are proved to exist for large, *energy-class* initial data (data with finite initial energy and nonnegative density) for the barotropic case in which the pressure is a power of density strictly greater than one (ideal gases are therefore excluded). Lions's results have been extended recently by Feireisl to the full nonbarotropic system with nonconstant viscosity and heat-conduction coefficients. See the treatises of Lions [11] and Feireisl [4], the references therein, and the currently most complete result in Feireisl [5]. The key breakthrough here is a quite deep analysis showing that sequences of approximate solutions, for which the only uniform estimates are the (physical) energy and entropy estimates, have *strongly* converging subsequences. The underlying analysis here is truly impressive and the class of initial data is surely close to optimal. On the other hand, solutions in this large-weak class possess so little regularity that analysis of their detailed qualitative properties seems very difficult. Moreover, these solutions are not unique, do not depend continuously on initial data, and in many cases are unphysical. Additionally, it is unknown whether these solutions obey the correct energy conservation principle, and in the nonbarotropic case the energy equation holds only in a very weak sense (again, see [5]). Finally, the functions of state and the viscosity and heat-conduction coefficients are quite restricted: ideal fluids are excluded in the nonbarotropic case, ideal isothermal flow is excluded in the barotropic case, and the coefficient functions predicted by Maxwell-Boltzmann theory cannot be accommodated. It is unclear right now how any of the qualitative properties of these large-weak solutions, or even their existence, should be interpreted physically.

The third category of solutions are in an *intermediate* regularity class, introduced in Hoff [6, 7, 8] for the case of constant viscosity and heat-conduction coefficients, in which the initial data is small in fairly weak norms, but not smooth: initial densities are in L^∞ and initial densities and velocities are in L^2; the L^2 norms must be small but not the L^∞ norm. These solutions can be discontinuous across hypersurfaces of $\mathbf{R}^n$, for example. Linearization is not valid in this class and solutions exhibit truly nonlinear and physically interesting effects, including the propagation of singularities and fronts. On the other hand, they exhibit just enough structure and regularity that their important features can be studied in a mathematically rigorous way. For example, there is a reasonable uniqueness and continuous dependence theory in Hoff [9], and piecewise smooth solutions are shown in Hoff–Santos [10] to satisfy the Rankine-Hugoniot conditions in a strict pointwise sense with strengths of jumps tracked quite explicitly in time.

Current research focuses on understanding better the *physical, qualitative* properties of solutions in the latter two classes so that the corresponding theories may eventually merge into a well-posedness theory for initial data in the correct regime of validity of the model, which so far has not been clearly established.

3.2 Vanishing Viscosity Method for Transonic Flow

We discuss some recent results and developments on subsonic-sonic and transonic flows past an obstacle via approximation methods.

In two significant papers written a decade apart, Morawetz [117, 119] presented a program for proving the existence of weak solutions to the equations governing two-dimensional steady irrotational inviscid compressible flow in a channel or exterior to an airfoil. As is well known, the classical results of Shiffman [127] and Bers [21, 22] apply when the upstream speed is sufficiently small, for which the flow remains subsonic and the governing equations are elliptic. However, beyond a certain speed at infinity (determined by the flow geometry), the flow becomes transonic which, coupled with nonlinearity, yields shock formation. Morawetz's program in [117, 119] was to imbed the problem within an assumed viscous framework for which the compensated compactness framework would be satisfied. Under this assumption, Morawetz proved that solutions of the as yet unidentified viscous problem have a convergent subsequence whose limit is a solution of the transonic flow problem.

In this talk, we present such a viscous formulation, hence completing part but not all of Morawetz's program. Specifically, a vanishing viscosity method is formulated for two-dimensional transonic steady irrotational compressible fluid flows with adiabatic constant $\gamma \in [1, 3)$ to ensure a family of invariant regions for the corresponding viscous problem, which implies an upper bound uniformly away from cavitation for the viscous approximate velocity fields. Mathematical entropy pairs are constructed through the Loewner-Morawetz relation via entropy generators governed by a generalized Tricomi equation of mixed elliptic-hyperbolic type, and the corresponding entropy dissipation measures are analyzed so that the viscous approximate solutions satisfy the compensated compactness framework. Then the method of compensated compactness is applied to show that a sequence of solutions to the viscous problem, staying uniformly away from stagnation with uniformly bounded velocity angles, converges to an entropy solution of the inviscid transonic flow problem. In this way, we can handle the case of cavitation points which had also challenged Morawetz, although the hypothesis of Morawetz about no stagnation points still remains in force.

For more details, see Chen-Dafermos-Slemrod-Wang [39] and Chen-Slemord-Wang [47].

3.3 Existence and Regularity of Solutions to Shock Reflection Problems

When a plane shock hits a wedge head on, it experiences a reflection-diffraction process and then a self-similar reflected shock moves outward as the original shock moves forward in time. Experimental, computational, and asymptotic analysis has shown that various patterns of shock reflection may occur, including regular and Mach reflection. However, most of the fundamental issues for shock reflection have not been understood yet, including the global structure, stability, and transition of the different patterns of shock reflection. Therefore, it is essential to establish the global existence and structural stability of solutions of shock reflection in order to understand fully the phenomena of shock reflection. On the other hand, there has been no rigorous mathematical result on the global existence and structural stability of shock reflection, including the case of potential flow which is widely used in aerodynamics. Such problems involve several challenging difficulties in the analysis of nonlinear partial differential equations including mixed equations of elliptic-hyperbolic type, free boundary problems, and corner singularity where an elliptic degenerate curve meets a free boundary.

In Chen-Feldman [43, 44], we have developed a rigorous mathematical approach to overcome these difficulties involved and established a global theory of existence and stability for shock reflection by large-angle wedges for potential flow. We have also studied optimal regularity of global regular reflection solutions by wedges. The techniques and ideas developed here will be useful to other nonlinear problems involving similar difficulties.

3.4 Thin Film Equation – Overcompressible Waves

Coating flows and their applications in physics, engineering, and biology have been the subject of decades of research. The mathematical study of these flows, i.e., of thin liquid films on solid substrates, begins with the lubrication approximation of the Stokes equations. The resulting partial differential equation, known as a thin film equation, is a nonlinear fourth-order equation for the height h of the free surface. Surface tension and gravity provide forces that generate flow in a variety of contexts, including spreading of droplets and layers of fluid on solid surfaces.

We consider the flow of a thin layer of fluid down an inclined plane, modified by the presence of surfactant. In this presentation, we consider only insoluble surfactant, whose transport and diffusion is restricted to the free surface, adding a partial differential equation for the surfactant concentration Γ. The equations in nondimensional form are

$$h_t + \left(\tfrac{1}{3}h^3\right)_x - \left(\tfrac{1}{2}h^2\Gamma_x\right)_x = \beta\left(\tfrac{1}{3}h^3 h_x\right)_x - \kappa\left(\tfrac{1}{3}h^3 h_{xxx}\right)_x, \tag{1}$$

$$\Gamma_t + \left(\tfrac{1}{2}h^2\Gamma\right)_x - \left(h\Gamma\Gamma_x\right)_x = \beta\left(\tfrac{1}{2}h^2\Gamma h_x\right)_x - \kappa\left(\tfrac{1}{2}h^2\Gamma h_{xxx}\right)_x + \delta\,\Gamma_{xx}. \tag{2}$$

This system has three small parameters, the coefficient κ of surface tension, the surfactant diffusivity δ, and the coefficient β of the gravity-driven diffusive spreading of the fluid. When all three parameters are zero, the nonlinear system of partial differential equations is hyperbolic/degenerate-parabolic. Then there is a one-parameter family of traveling waves in which h is piecewise constant, and Γ is continuous, piecewise linear, and zero outside a bounded interval. This family is overcompressive in the sense that small perturbations ahead of or behind the wave propagate towards the wave. The corresponding family of solutions with non-zero values of the physical parameters is investigated using perturbation theory. Details of the thin film equations, and the study reported in the presentation at Banff, may be found in the references [14, 15, 16, 17].

4 Scientific Progress Made

The related results from panel discussion can be summarized and grouped under a number of topics as follows.

Subsonic-Sonic and Transonic Flows past an Obstacle: The panel focused on recent results presented in various articles (we refer the reader to the articles by Chen-Dafermos-Slemrod-Wang in [39], Chen-Slemrod-Wang [47], and the references therein), where the equations of planar compressible gas dynamics for steady irrotational isentropic (or isothermal) flow around an obstacle were considered. It is well known from the classic work of Shiffman and later Bers in the 1950's that, for fluid speed q_∞ at infinity less than some critical speed $q*$ the steady equations admit smooth solutions. When q_∞ reaches $q*$, at some place the flow is going from subsonic (elliptic) to sonic-subsonic (degenerate elliptic). In [39], a simple and elegant resolution of this problem was presented via the method of compensated compactness. The main idea is to realize that the equations for conservation of linear momentum yield the crucial entropy-entropy flux pairs that make compensated compactness work. The question remains what happens when q_∞ exceeds q*. In this case, the flow will become completely transonic giving rise to a mixed hyperbolic-elliptic initial value problem. Morawetz had shown that if there exists a suitable viscous regularization to the steady gas dynamics equations which provides good uniform bounds on the solution of the viscous problem, then passage to an inviscid limit could be accomplished via the method of compensated compactness. But up to now Morawetz's program remained blocked by the lack of such a good viscous system. In [47], a good viscous formulation to the steady gas dynamics equations was given which assisted deriving the desired bounds and estimates for the adiabatic constant $\gamma \in [1, 3)$. The case of cavitation points, which had also challenged Morawetz, has been handled although the hypothesis of Morawetz about no stagnation points still remains in force.

Shock Reflection: Shock reflection is one of the most important open problems in mathematical fluid mechanics. The regular reflection problem has been studied extensively for some simplified models of the Euler equations and real breakthroughs have been made. In particular, the global existence and stability of solutions to regular shock reflection were established for potential flow when the wedge has a large angle in Chen-Feldman [43, 44]. Also see Canic *et al.* [27, 28, 93] for the unsteady transonic small disturbance equation (UTSD) and a nonlinear wave system, and Zheng [155] for the pressure-gradient system for related results. These studies have provided important ideas and techniques for solving the shock reflection problem for the full Euler equations. Hunter *et al.* [89, 90, 139] found numerical evidence of a new phenomenon in shock reflection (Guderley reflection).

Transonic Shocks and Supersonic Flow: In Chen-Feldman [40, 41, 42, 45], the existence and stability of steady multidimensional transonic shocks for potential flow was proved under a steady perturbation of the upstream uniform supersonic flow; and the existence and stability of multi-dimensional transonic flows through an infinite nozzle of arbitrary cross section was established. In Liu *et al.* [65, 66], it is shown, for the potential flow equation, that a self-similar solution does not develop a sonic/supersonic bubble with a region of subsonic flow; and that when a wedge of not too large angle accelerates from zero to the supersonic speed, it is the weak shock that appear in the time-asymptotic state. This answers the Prandtl conjecture on supersonic flow passing a wedge.

Other Results: In [49], the global structure of solutions with triple shocks to the generalized Riemann problems of two-dimensional simplified compressible Euler equations are obtained. Computations of certain two-dimensional flows in special setups were performed in Shu-Wang *et al.* [54] and Shu-Zheng *et al.* [130] to help understand certain specific features of the flow with the objective of helping the analysis. In [155], a derivation of the pressure-gradient system was presented by Hunter-Zheng. In Wang-Zheng [144], a Goursat problem for two-dimensional wave interactions of Riemann solutions is studied. Additional related results were obtained in [24, 29, 31, 32, 33, 50, **?**, 61, 62, 64, 69, 70, 86, 94, 102, 103, 129, 132, 134, 143, 154] and so on, and surveyed by Slemrod in SIAM News: 39 (5), June 2006, page 3.

5 Outcome of the Meeting

Here are some remarks. First we came to realize the key role of mixed hyperbolic-elliptic systems of partial differential equations. This was no surprise since these problems arise naturally in both shock reflection and transonic flow in fluid mechanics and in isometric embedding problems in differential geometry. Hence we plan to create a research program emphasizing mixed problems both in their classical role in compressible fluid flow and in their applications in bio-mechanics and differential geometry. We think that this is an area ripe for breakthroughs on questions that had often been left in research monographs in the category of "unsolved open problems" (e.g., Bers [22], page 135; Courant-Friedrichs [57], page 317; Lax [99], page 427; Morawetz [120], page 24; Yau [149], page 355). The physical geometry of our fluid problems seems to be playing a major role in our analysis. For example, special small scale effects of airfoil shape seem to be playing a crucial role.

6 Workshop Program

October 29, Sunday

9:00-10:00: Survey Talk: Tai-Ping Liu
10:05-10:45: Mikhail Feldman

Break

11:00-11:30: Michael Westdickenberg
11:35-12:05: Cleopatra Christoforou

Lunch and Discussion

3:00-3:40: Yuxi Zheng
3:45-4:15: Tao Luo

Break

4:30-5:00: Kris Jenssen
5:05-5:35: Volker Elling

October 30, Monday

8:30-9:30: Survey Talk: David Hoff
9:35-10:15: Suncica Canic

Break

10:40-11:20: Helge Holden
11:25-12:05: Athanasios Tzavaras

Lunch and Discussion

2:00-2:40: Dietmar Kroener
2:45-3:15: Monica Torres

Break

3:40-4:20: Pierre-Emmanuel Jabin
4:25-4:55: Dianwen Zhu
5:00-5:30: Razvan Fetecau

October 31, Tuesday

8:30-9:30: Survey Talk: Marshall Slemrod
9:35-10:15: Yongqian Zhang

Break

10:30-11:40: Robert McCann
11:15-11:45: Ronghua Pan
11:50-12:20: Laura Spinolo

Lunch and Discussion

Free Discussion

8:00-9:30pm: Panel Discussion
Chair: Constantine Dafermos and Barbara Keyfitz

November 1, Wednesday

8:30-9:30: Survey Talk: Michael Shearer

9:35-10:15: Christian Klingenberg

Break

10:45-11:25: Dehua Wang
11:30-12:10: Tong Li

Lunch and Discussion

2:30-3:30: Survey Talk: Walter Craig
3:15-3:55: Hermano Frid

Break

4:10-4:50: Michel Rascle
4:55-5:35: Fabio Ancona

November 2, Thursday

9-11:30: Free Discussion

11:30: End of Workshop

7 List of Participants

Ancona, Fabio: University of Bologna, Italy
Canic, Suncica: University of Houston, USA
Chen, Gui-Qiang: Northwestern University, USA
Christoforou, Cleopatra: Northwestern University, USA
Craig, Walter: McMaster University, Canada
Dafermos, Constantine: Brown University, USA
Elling, Volker: Brown University, USA
Feldman, Mikhail: University of Wisconsin, USA
Fetecau, Razvan: Simon Fraser University, Canada
Frid, Hermano: IMPA, Brazil
Ghoussoub, Nassif: Banff International Research Station, Canada
Hoff, David: Indiana University, USA
Holden, Helge: NTNU at Trondheim, Norway
Jabin, Pierre-Emmanuel: University of Nice, France
Jenssen, Kris: Pennsylvania State University, USA
Keyfitz, Barbara Lee: Fields Institute, Canada, and University of Houston, USA
Kim, Eun Heui: California State University at Long Beach, USA
Klingenberg, Christian: University of Wuerzburg, Germany
Kroener, Dietmar: University of Freiburg, Germany
Li, Tong: University of Iowa, USA
Liu, Tai-Ping: Stanford University, USA
Luo, Tao: Georgetown University, USA
McCann, Robert: University of Toronto, Canada
Pan, Ronghua: Georgia Institute of Technology, USA
Panferov, Vladislav: McMaster University, Canada
Rascle, Michel: University of Nice, France

Shearer, Michael: North Carolina State University, USA
Slemrod, Marshall: University of Wisconsin at Madison, USA
Sospedra-Alfonso, Reinel: University of Victoria, Canada
Spinolo, Laura Valentina: Northwestern University, USA
Torres, Monica: Purdue University, USA
Trivisa, Konstantina: University of Maryland, USA
Tzavaras, Athanasios: University of Maryland, USA
Westdickenberg, Michael: Rheinische Friedrich-Wilhelms-Universitaet, Germany
Zhang, Yongqian: Fudan University, PRC
Zheng, Yuxi: Pennsylvania State University, USA
Zhu, Dianwen: University of Maryland, USA

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