

The Workshop will concentrate on research and goals in three main subjects – **resolution of singularities, factorization of birational mappings, and toroidalization of morphisms** – as well as on interactions among them.

The morning sessions will be devoted mainly to mini-series of three lectures each on the topics above, to be given by Edward Bierstone, Kalle Karu and Dale Cutkosky (respectively). The first day will begin with an Overview, by Dan Abramovich. The working afternoons will include invited talks by other participants; we plan to post titles and abstracts shortly.

Tuesday afternoon will be free, for discussion or for exploration of the beautiful areas around BIRS. We plan to schedule no formal talks on Thursday, but will encourage informal discussion during the morning.

Resolution of Singularities. The study of singularities in algebraic and analytic geometry is a fast-developing field of research, having applications in many areas of mathematics and physics. The problem of resolution of singularities has a long and outstanding history starting with Newton and continuing through the twentieth century in the fundamental work of Albanese, Zariski, Hironaka, etc. To quote Lipman’s *Featured Math. Review* MR 98e:14010, “Hironaka’s theorem on the existence of resolution of singularities is an outstanding achievement of twentieth-century mathematics, by virtue of the depth both of its proof and its applications ... [But] Hironaka’s proof is lengthy, difficult, and non-constructive. Influential as the proof has been, few people can have checked it through entirely. ... Simplified, more algorithmic proofs are important not only for imparting better understanding of what is really involved in this great theorem, but also for their potential value in unearthing basic features of singularities and their classification.”

The algorithms for canonical desingularization in characteristic zero by Bierstone-Milman and Villamayor brought a fundamental simplification in our understanding of resolution of singularities, in the 1990’s. There has since been an explosion of new activities that will be discussed in the Workshop.

Factorization of Birational Mappings. The classical “*weak factorization conjecture*” is the following: A proper birational map between complete nonsingular varieties over a field k can be decomposed into a sequence of blowings-up and blowings-down with nonsingular centers. Recently, S.D. Cutkosky solved a local version of the conjecture, while Włodarczyk and Abramovich-Karu-Matsuki-Włodarczyk settled the global version, both in characteristic 0. The essential points of their methods are:

(a) *Monomialization of morphisms.* Given a morphism $f : X \rightarrow Y$, find sequences of blowings-up of the source and target with smooth centers, after which the induced morphism $f' : X' \rightarrow Y'$ can be expressed locally by monomials with respect to appropriate coordinates.

(b) *Birational cobordism.* Initiated by Włodarczyk, the theory may be regarded as an algebraic version of Morse theory.

One of the main problems in the area is the *strong factorization conjecture*: A proper birational map between complete nonsingular varieties can be decomposed into a sequence of blowings-up followed by a sequence of blowings-down. A better understanding of monomialization and birational cobordism techniques, and especially of the relationship between these methods, may hold a key to strong factorization. These techniques will be a focus of our workshop.

Toroidalization of Morphisms. The *toroidalization conjecture* asserts the following: Let $f : X \rightarrow Y$ be a morphism of complete nonsingular varieties over a field of characteristic zero. Then there exist sequences of blowings-up of X and Y with smooth centres so that the resulting varieties X' and Y' have toroidal structures and the induced morphism $f' : X' \rightarrow Y'$ is toroidal.

First raised by S. Akbulut and H. King as a tool for understanding the topology of families of varieties, a weak version of this conjecture was proved by Abramovich and Karu on route to proving weak semi-stable reduction. The conjecture can be regarded as resolution of singularities of a mapping or, more precisely, transformation of a mapping by blowings-up of the source and target to a mapping that is “logarithmically smooth” – smooth after taking logarithms.

Monomialization (as above) is a local version of toroidalization. Cutkosky’s recent solutions in the cases $\dim X = 3$ and $\dim Y = 2$ or 3 are important steps. An affirmative solution of the toroidalization conjecture would reduce strong factorization of general birational maps to the toric case. Since toroidalization is a problem of resolution of singularities, a better understanding of canonical and constructive algorithms for desingularization can be expected to play a key role.