BIRS Workshop
Semimartingale Theory and Practice in Finance
June 5 - June 10, 2004

MEALS

Breakfast (Continental): 7:00 - 9:00 am, 2nd floor lounge, Corbett Hall, Sunday - Thursday
*Lunch (Buffet): 11:30 am - 1:30 pm, Donald Cameron Hall, Sunday - Thursday
*Dinner (Buffet): 5:30 - 7:30 pm, Donald Cameron Hall, Saturday - Wednesday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each lunch and dinner.**

MEETING ROOMS

All lectures are held in the main lecture hall, Max Bell 159. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155-159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

SCHEDULE (PRELIMINARY VERSION)

	Sat	Sunday	Monday	Tuesday	Wednesday	Thursday
7:00-9:00	X	Continental Breakfast, 2nd floor lounge, Corbett Hall				
9:00–9:45	X	Musiela	Davison	Becherer	Guo	Grasselli
9:45-10:30	X	Hobson	Albanese	Frittelli	Labbé	Carmona
10:30-11:15	X	Coffee Break, 2nd floor lounge, Corbett Hall				
11:15-12:00	X	Cont	Schachermayer	Tourin[1]	Pistorius	Berndt
12:00-14:00	X	Buffet Lunch, Donald Cameron Hall[2]				
14:00-14:45	X	Eberlein	Zervos	free afternoon	Melnikov	X
14:45-15:30	X	Tompaidis	Shreve	free afternoon	Karatzas	X
15:30-16:00	X	Coffee Break, 2nd floor lounge, Corbett Hall (except Tues.)				X
16:00-16:45	X	Rafailidis	Henderson	free afternoon	TBA	X
16:45-17:30	X	X	X	free afternoon	X	X
17:30-19:30	Buffet Dinner, Donald Cameron Hall[3]					X

[1]This talk will start at 11:00. A group photo will be taken at 11:45 am, directly after the talk. Please meet on the front steps of Corbett Hall.

[2]A free guided tour of The Banff Centre is offered to all participants and their guests on Sunday starting at 1:00 pm. The tour takes approximately 1 hour. Please meet in the 2nd floor lounge in Corbett Hall.

[3]At 5:30 pm on Saturday evening we will meet for predinner drinks in Props Pub beside Donald Cameron Hall.

Some Suggestions:

- Warm Dress: remember Banff is relatively high in the mountains, and evenings will likely be cold. So please bring a warm jacket. Mountain shoes or boots are also highly recommended.
- Dress code: we suggest casual/informal dress throughout the week.
- When you arrive on Saturday, please check in at the Professional Development Centre at the Banff Centre. We are assured your rooms will be available by 4:00 pm on Saturday. You may be able to move in earlier if you ask at check in.
- Depending on availability, it may be possible to stay one extra night (Thursday) at BIRS for a nominal charge. Contact Andrea Lindquist at 403 763 6999 immediately if you are interested.
- There will be jazz at the Banff Centre during the week as part of the "Jazz at Banff" festival. See http://www.banffcentre.ca/events/music/jazz/ for programme details.

Talks: You may use blackboard, overhead projection or computer projection. If you intend to use the LCD projector, it should be sufficient to bring your talk on a USB stick or CDRom.

Computers: The BIRS computing environment consists of two networks, one for thin- client "SunRay" terminals, and the other for guest's laptops. The SunRays are 15" LCD panels with a keyboard, mouse and network connection. They have no moving parts, so are completely silent. They connect to a central Sun server that runs Solaris 2.9, and the GNOME 2.0 desktop environment. The full suite of standard UNIX software is installed, as well as the most popular tools for mathematics and academic publishing. The high-speed, permanent Internet connection and a robust selection of Internet software is always available via these terminals.

The guest network for laptops is a fast ethernet network, with ports available in all bedrooms, in most meeting areas, and in the main seminar room. Configuration for network access is done automatically via DHCP."

Library: BIRS has no physical library. It has online access to MathSciNet. Participants who have online services at their home institutions are encouraged to establish proxie accounts which will allow them journal access from BIRS.

The Banff Centre has a very nice arts/music library which has a good collection of sheet music as well as recorded music and video material. BIRS participants are welcome to use this library.

The Banff Centre: The Banff Centre is a centre for fine arts programmes, leadership development, mountain culture, and now also BIRS. It is situated in the town of Banff part way up Tunnel Mountain in a peaceful and secluded site.

It has an athletic complex with swimming pool, running track, workout room, climbing wall etc. all of which are available to BIRS participants. For more on the Banff Centre and its current artistic programme, please see the Banff Centre webpage.

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ABSTRACTS
(in alphabetic order by talks surname)

Speaker: **Claudio Albanese** (Imperial College London)
Title: *Unifying Volatility Models*
Abstract: This presentation reviews analytical methodsfor building analytically tractable option pricing models that combine state-dependent volatility, stochastic volatility and jumps.Spectral analysismethods are used to add jumps and stochastic volatility to hypergeometric Brownian motions.A Poisson approximation scheme for jump processesis used it to construct numerical discretizations for the corresponding partial integro-differential equations. Transition probabilities are computed analytically as expansions in orthogonal polynomials to ensure that results dont depend on the size of time steps and to maintain local no-arbitrage conditions. Finally, stochastic volatility can be accounted for by means of a new class of affine lattice models. Wereview applications to equity options, interest rate derivative models with stochastic volatility, credit derivative models and convertible bonds.

Collaborators: Oliver Chen, Alexey Kuznetzov, Stephan Lawi.

Speaker: **Dirk Becherer** (Imperial College London)
Title: *On general futures prices in supermartingale term structure models*
Abstract: Futures are among the practically most important and liquidly traded instruments. There is a vast literature on them with many articles, and also excellent treatment in textbooks on mathematical finance by Björk, Duffie, and Hunt & Kennedy, among others. Nevertheless, the literature about theoretical models on futures appears technically rather less developed and mathematically elaborated than other areas of mathematical finance, like equities, credit risk, or interest rate theory.

This talk is concerned with general futures prices in framework of semimartingale pricing kernel models with positive interest, i.e. supermartingale pricing models. We show how recent results on supermartingale term structure models in connection with stochastic backward integration allow for a general and unifying view on discretely and continuously resettled contingent claims, and also reveal the natural martingale structure behind the appropriate numeraires for obtaining the futures price process. The analysis does not need to assume a Brownian filtration or continuous martingales. And for key results, one does not need to assume the existence of a classical savings account either.

Speaker: **Antje Berndt** (Cornell University)
Title: *Measuring default risk premia from default swap rates and EDFs*
Abstract: This paper estimates recent default risk premia for U.S. corporate debt, based on a close relationship between default probabilities, as estimated by the Moody's KMV EDF measure, and market default swap (CDS) rates. The default-swap data, obtained by CIBC from a large number of dealers and bank counterparties, allow us to establish a strong link between actual and risk-neutral default probabilities for

69 firms in the three sectors that we analyzed: broadcasting and entertainment, healthcare, and oil and gas. This is joint work with Rohan Douglas, Darrell Duffie, Mark Ferguson, and David Schranz.

Speaker: **Rene Carmona** (Princeton University)
Title: *TBA*
Abstract: TBA

Speaker: **Patrick Cheridito** (Princeton University)
Title: *Superhedging European contingent claims under gamma constraints* (joint work with Mete Soner and Nizar Touzi)
Abstract: The classical Black-Scholes strategy of a European contingent claim may require rapid changes in the replicating portfolio. One approach to avoid this is to impose a priori bounds on the variations of the allowed trading strategies, called gamma constraints. Under such a restriction, it is in general no longer possible to replicate a European contingent claim, but one can still ask one-self the question what is the infimum of all initial capitals that allow an investor to super-replicate the contingent claim. It is shown that this infimum is the unique viscosity solution to a non-standard pde. The derivation of the viscosity property is based on new results on the small time path behavior of double stochastic integrals.

Speaker: **Rama Cont** (Ecole Polytechnique, France)
Title: *Integro-differential equations and finite difference methods for option pricing in models based on Lévy processes*
Abstract:

In stochastic models where the random evolution of the underlying asset is driven by a Lvy process or a time-inhomogeneous jump-diffusion process, European and barrier options in models with jumps are formally expressible as solution of partial integro-differential equations. We study the precise relation between such partial integro-differential equations (PIDEs) and the values of European and barrier options in exponential Lvy models. After giving sufficient conditions under which options prices are classical solutions of the PIDEs, we illustrate that these conditions may fail in pure-jump models where the option prices can have non-smooth dependence on the underlying. We give sufficient conditions on the Lvy triplet for the option price to be continuous in the underlying asset and show that in this case it is the unique viscosity solution of the PIDE, using an appropriate extension of the notion of viscosity solution to such nonlocal boundary value problems. This notion of solution, though lacking smoothness, is suitable for studying convergence of numerical schemes: we propose an explicit-implicit time-stepping scheme to solve the equation and study its stability and convergence. We also discuss localization to a finite domain and provide an estimate for the localization error under an integrability condition on the Levy measure. Numerical tests are performed with smooth and non-smooth initial conditions. Our scheme can be used for European and barrier options, applies in the case of pure-jump models or degenerate diffusion coefficients, and extends to time-dependent coefficients.

Speaker: **Matt Davison** (University of Western Ontario)
Title: *Fractional Brownian motion and the optimization of hydroelectric facilities*
Abstract: A problem of current industrial interest is the optimal operation of a hydroelectric facility in deregulated electricity markets. This problem is treated in [1] using ideas of real options theory. Unless a simple pump storage hydro system is treated, such a stochastic optimization must be done in the presence of both random prices and random water inflows. While [1] allows a fairly wide class of stochastic price models to be treated, it assumes Brownian motion based randomness for water inflows. However, since Hurst it has been known that river flows are not simple diffusions but have long range memory. This long-range memory can be characterized by the Hurst exponent, ? for Brownian motions and ¿ ? for long memory processes. The hydrology literature suggests this long-range memory will be even more important for the short time scales and relatively small watershed scales of hydroelectric modeling.

Perhaps the simplest way to create a random process with Hurst exponent ¿ ? is to use fractional Brownian motion, the stochastic calculus of which is reviewed in [2]. In recent years this family of processes

has attracted a certain amount of interest by the financial math community, and the associated idea of Wickbitrage has generated some controversy, see [3] and references therein.

Speaker: **Ernst Eberlein** (University of Freiburg)
Title: *Pricing interest rate derivatives in a Levy term structure model*
Abstract: Models driven by Levy processes are attractive in finance since due to their greater flexibility they reduce model risk. First we discuss a generalization of the default-free Levy term structure model which was developed in [1] and [2]. In this framework European options can be priced explicitly. Then we develop a model for LIBOR rates based on Levy processes and derive conditions for arbitrage-freeness. An explicit formula to price caps and floors is given. This formula which is based on bilateral Laplace transforms allows fast numerical evaluation.

References

[1] E. Eberlein and S. Raible: Term structure models driven by general Levy processes. Mathematical Finance 9 (1999), 31-53

[2] E. Eberlein and F. Özkan: The defaultable Levy term structure: ratings and restructuring. Mathematical Finance 13 (2003), 277-300

[3] E. Eberlein and F. Özkan: The Levy Libor model. FDM Preprint 82 (2002)

[4] E. Eberlein, J. Jacod and S. Raible: Levy term structure models: no-arbitrage and completeness. FDM Preprint 83 (2003)

Speaker: **Marco Frittelli** (Universitá di Firenze)
Title: *Title*
Abstract: When the price processes of the financial assets are described by possibly unbounded semimartingales, the classical concept of admissible trading strategies may lead to a trivial utility maximization problem because the set of bounded from below stochastic integrals may be reduced to the zero process. However, it could happen that the investor is willing to trade in such a risky market, where potential losses are unlimited, in order to increase his/her expected utility. We translate this attitude into mathematical terms by employing a class H^W of W-admissible trading strategies which depend on a loss random variable W. These strategies enjoy good mathematical properties and the losses they could give rise to in the trading are compatible with the preferences of the agent. We formulate and analyze by duality methods the utility maximization problem on the new domain H^W. We show that, for all loss variables W contained in a properly identified set W, the optimal value on the class H^W is constant and coincides with the optimal value of the maximization problem over a larger domain K_Φ. The class K_Φ doesn't depend on the single W?W, but it depends on the utility function u through its conjugate function Φ. By duality methods we show that the optimal solution exists in K_Φ and it can be represented as a stochastic integral that is a uniformly integrable martingale under the minimax measure. We provide the economic interpretation of the larger class K_Φ and we analyze some examples that show that this enlargement of the class of trading strategies is indeed necessary.

Speaker: **Matheus Grasselli** (McMaster University)
Title: *TBA*
Abstract: TBA

Speaker: **Xin Guo** (Cornell University)
Title: *Optimal Partially Reversible Investment with Entry Decision and General Production Function*
Abstract: We study the problem of a company that adjusts its stochastic production capacity in reversible investments by purchasing capital at a given price and selling capital at a lower price. The company may also decide on the activation time of its production. The profit production function is of a very general form satisfying minimal standard assumptions. The objective of the company is to find an optimal entry and production decision to maximize its expected total net profit over an infinite time horizon. The resulting dynamic programming principle is a two-step formulation of a singular stochastic control problem

and an optimal stopping problem. Our analysis is based on viscosity solutions of the associated Bellman variational inequalities. We provide a complete solution with explicit expressions of the value functions and the optimal controls: the company activates its production once a fixed entry-threshold of the capacity is reached, and invests in capital so as to maintain its capacity in a closed bounded interval. The boundaries of these regions can be computed explicitly and their behavior are studied in terms of the parameters of the model.

Joint work with H. Pham of Paris 7.

Speaker: **Vicky Henderson** (Princeton University)
Title: *Valuing Real Options without a Perfect Spanning Asset*
Abstract: The real options approach to corporate investment decision making recognizes a firm can delay an investment decision. The well known models of McDonald and Siegel (1986) and Dixit and Pindyck (1994) value the investment decision as a perpetual American call option on the project value. The former assumes an equilibrium framework whilst the latter, along with much of the literature, uses a replicating portfolio argument and thus a complete model.

We instead assume only a partial spanning asset can be found which is imperfectly correlated with project value. This is more realistic, as most real projects can only be partially hedged by traded securities and private risks are common. Both the equilibrium and complete models are special cases of our model, obtained when risk aversion tends to zero and when correlation approaches one, respectively. The equilibrium valuation also coincides with valuation under the minimal martingale measure.

Under the partial spanning model, the value of the option to invest and the optimal stopping (investment) time are derived in closed form, requiring consideration of time consistency properties of utility functions. In our model, the value of the option to invest and the trigger level are both lowered when the spanning asset is less than perfect. This implies the firm should invest earlier than the classic real options models suggest. In fact, as correlation tends to one, there is no continuity of the critical value of a parameter, above which a non-trivial solution exists. This means there are situations where our model (with a less than perfectly correlated traded asset) recommends investment at a certain finite trigger level, but under the equilibrium and perfect spanning models, the firm should never invest. Thus, by taking private risks into account, the partial spanning model gives a much richer model of corporate investment decisions.

Speaker: **David Hobson** (University of Bath)
Title: *TBA*
Abstract: TBA

Speaker: **Ioannis Karatzas** (Columbia University)
Title: *Diversity and Arbitrage in Equity Markets*
Abstract: An equity market is called "diverse" if no single stock is ever allowed to dominate the entire market in terms of relative capitalization. In the context of the standard Ito-process model initiated by Samuelson (1965) we formulate this property (and the allied, successively weaker notions of "weak diversity" and "asymptotic weak diversity") in precise terms. We show that diversity is possible to achieve, but delicate. Several examples are provided which illustrate these notions and show that weakly-diverse markets contain relative arbitrage opportunities: it is possible to outperform or underperform such markets over any given time-horizon. The existence of this type of relative arbitrage does not interfere with the development of contigent claim valuation, and has consequences for the pricing of long-term warrants and for put-call parity. Several open questions are suggested for further study. (Joint work with R. Fernholz and C. Kardaras.)

Speaker: **Chantal Labbé** (University of Waterloo)
Title: *TBA*
Abstract: TBA

Speaker: **Alexander Melnikov** (University of Alberta)
Title: *Efficient hedging and equity–linked life insurance*
Abstract: The talk is devoted to how hedging methodologies developed in the modern financial mathematics can be exploited to price equity-linked life insurance contracts. We study pure endowment life insurance contracts with fixed and flexible guarantees. In our setting, these insurance instruments are based on two risky assets of the market controlled by Black-Scholes model during a contract period. The first asset is responsible for the maximal size of a future profit while the second, more reliable, asset provides a flexible guarantee for the insured. The insurance company is considered as a hedger of a maximum of these assets conditioned by remaining life time of a client in the framework of this market. The main attention is paid to new types of hedging (quantile hedging and efficient hedging with power loss function), which, together with Black-Scholes (fixed guarantee) and Margrabe (flexible guarantee) formulae, creates effective actuarial analysis of such contracts. We show also how this approach is extended to a jump-diffusion scheme and discuss some connections with the pricing of credit risks and defaultable derivative securities. Finally, we give numerical examples based on financial indices the Dow Johes Industrial Average and the Russell 2000 to demonstrate how our results can be applied to actuarial practice.

Speaker: **Marek Musiela** (BNP Paribas)
Title: *Mathematical and practical issues with volatility modelling (joint work with Pierre-Louis Lions)*
Abstract: The are many ways to define a model which fits the market smile. The problem is more how to choose one among many possible solutions. The talk will concentrate on how some natural mathematical and practical constraints can serve as a guide. Particular attention will be given to the role played by correlation.

Speaker: **Martijn Pistorius** (Kings College, London)
Title: *Pricing of American, Russian and barrier options under exponential phase type Levy models*

Abstract: The search for a jump diffusion or pure jump model to outperform the Black-Scholes model was initiated by Mandelbrot and Fama followed by Merton, with the jump-diffusion with Gaussian jumps, and continues nowadays in the work of Carr, Chang, Madan, Geman and Yor who propose the variance-gamma model, of Eberlein who proposes the hyperbolic model, of Barndorff-Nielsen with the normal inverse Gaussian model, of Kou who proposed a jump-diffusion with exponential jumps and of Koponen who introduced the Koponen family. In our talk we adresses only the issue of the analytical tractability of pricing certain perpetual American or barrier options. To that end we examine a jump-diffusion model where the jumps form a compound Poisson process with jump distribution of phase–type. On the one hand this phase–type model is quite rich, since this class of processes is known to be dense in the class of all Lévy processes, and on the other hand for many options the model is analytically tractable.

We illustrate this in the case of the American put option and the Russian option.

Speaker: **Walter Schachermayer** (Vienna University of Technology)
Title: *On Utility Based Pricing of Contingent Claims in Incomplete Markets.* with J. Hugonnier and D. Kramkov
Abstract: We study the uniqueness of the utility based price of contingent claims in a semimartingale model of incomplete financial market. In particular, we obtain that a necessary and sufficient condition for all bounded contingent claims to admit a unique utility based price is that the solution of the dual problem defines an equivalent martingale measure.

Speaker: **Steven E. Shreve** (Carnegie Mellon University)
Title: *A Two-Person Game for Pricing Convertible Bonds*
Abstract: A firm issues a convertible bond. At each subsequent time, the bondholder must decide whether to keep the bond, thereby collecting coupons. or to convert the bond for stock. The bondholder wishes to choose a conversion strategy to maximize the bond value. Subject to some restrictions, the bond can be

called by the issuing firm, which acts to maximize equity value and thus minimize bond value. This creates a two-person game, and we model the bond price as the value of this game. We show, however, that under the assumption that dividends are paid at a lower rate than the short-term interest rate, this game reduces to one of two optimal stopping problems, and which is the relevant problem can be determined apriori.

Because the dividends paid depend on the value of equity, which in turn depends on the value of the bond, the dynamics of the firm value cannot be specified until the bond pricing problem is solved. As a result, the optimal stopping problems that must be solved lead to nonlinear partial differential equations. These can be solved by a fixed-point method.

This work is Mihai Sirbu's Ph.D. dissertation.

Speakers: **Stathis Tompaidis** (University of Texas, Austin)
Title: *Efficient Computation of Hedging Parameters for Discretely Exercisable Options*
Abstract: We describe a method to obtain bounds on the hedging parameters of discretely exercisable options using Monte-Carlo simulation. The method is based on a combination of the duality formulation of the optimal stoping problem for pricing discretely exercisable options and Monte-Carlo estimation of hedging parameters for European options. For a given computer budget and exercise strategy we provide an algorithm that achieves the tightest bounds. The method can handle arbitrary payoff functions, general diffusion processes, and a large number of random factors. We also present a fast, heuristic, alternative method and use our method to evaluate its accuracy.

Speaker: **Agnes Tourin** (McMaster University)
Title: *Splitting methods for Hamilton-Jacobi-Bellman equations arising in mathematical finance.*
Abstract: Many problems arising in Finance, such as Portfolio Selection in incomplete markets, are adequately modelled by the theory of stochastic control. It is often convenient to solve the problem by approximating numerically the Hamilton-Jacobi-Bellman equation characterizing the optimal policies. Monotone finite difference schemes for Hamilton-Jacobi equations based on the framework developed by Barles and Souganidis will be discussed. In particular, I will explain how the exploitation of several kinds of operator splitting methods leads to simple numerical schemes. An application will be presented.

Speaker: **Mihail Zervos** (Kings College London)
Title: *Pricing a class of exotic options via moments and SDP relaxations*
Abstract: We present a new methodology for the numerical pricing of a class of exotic derivatives such as Asian or barrier options when the underlying asset price dynamics are modelled by a geometric Brownian motion or a number of mean-reverting processes of interest. This methodology identifies derivative prices with infinite-dimensional linear programming problems involving the moments of appropriate measures, and then develops suitable finite-dimensional relaxations that take the form of semi-definite programs indexed by the number of moments involved. By maximising or minimising appropriate criteria, monotone sequences of both upper and lower bounds are obtained. Numerical investigation shows that very good results are obtained with only a small number of moments. Theoretical convergence results are also established.