ABSTRACTS

Mean curvature flow with free boundary on smooth hypersurfaces

In this talk we consider the classical mean curvature flow of hypersurfaces with boundary satisfying a (homogeneous) Neumann condition on an arbitrary, fixed, smooth hypersurface in Euclidean space. In particular, we focus on the problem of singularity formation on the free-boundary and the classification of the limiting behaviour thereof. By developing a monotonicity formula analogous to that of Huisken for the boundaryless case, we show that any smooth blow-up centered about a boundary point is self-similar.

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Isoperimetric Estimate for the Ricci Flow on $S^2 \times S^1$

In this talk, we study the dilation limit of $S^2 \times S^1$ with a warped product metric under the Ricci flow. We first prove that the isoperimetric ratio on the base manifold S^2 has a lower bound, which excludes the $\Sigma^2 \times \mathbb{R}$ as the dilation limit. We also prove a monotonicity result under a certain condition.

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Uniformization of non-compact gradient Kaehler Ricci solitons with positive curvature

In this talk we present the result that if M is a non-compact gradient Kaehler- Ricci soliton which is either a) expanding with non-negative Ricci curvature, or b) steady with positive Ricci curvature and such that the scalar curvature attains its maximum on M, then M is biholomorphic to C^n, complex Euclidean space. We will also discuss our results and techniques in the context of the uniformization conjecture of Greene-Wu-Yau; that all complete non-compact Kahler manifolds with positive holomorphic bisectional curvature are biholomorphic to C^n.

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The application of geometry at infinity to the Ricci flow

I would like to give several applications of geometry at infinity to the Ricci flow. In particular, I will focus on 2-dimensional and 3-dimensional cases.

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Some Results and Open Problems on Ricci Solitons

Ricci solitons are solutions of the Ricci flow that are stationary up to diffeomorphism and homothety. The basic features of solitons will be explained, including elementary examples and the connection with the Harnack inequality for the Ricci flow. Known results about the existence of compact solitons and convergence to solitons will be reviewed, and the talk will end with a discussion of some open problems.

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Precise asymptotics of the Ricci flow neckpinch.

Local singularities of Ricci flow were first rigorously constructed by M. Simon. The most natural way a local singularity can form, at least in low dimensions, is by pinching a nearly cylindrical neck. This talk will describe ongoing work with S. Angenent. Our earlier work constructs neckpinch singularities on compact manifolds and derives a priori estimates for their formation. Our recent results show that these estimates are sharp, and provide precise asymptotics for developing neckpinch singularities in all dimensions $n \geq 3$.

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Mixed volume preserving curvature flows

We consider a class of fully nonlinear, parabolic evolution equations for compact, strictly convex hypersurfaces in Euclidean space. The speed of the evolving surfaces consists of a function which is positive, monotone, homogeneous of degree one in the principal curvatures, balanced by a global term which fixes any particular mixed volume under the flow. We show that the evolution has a smooth solution for all time which converges exponentially to a sphere. This work generalizes the author's earlier results for mixed volume preserving mean curvature flows. Special cases of the new flow can be used to re-prove the Minkowski inequalities of convex geometry.

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Convergence of the Ricci flow to the solitons

We will consider a τ-flow, given by $\frac{d}{dt}g_{ij} = -2R_{ij} + \frac{1}{\tau}g_{ij}$, where $\tau > 0$. If the flow exists for all times $t > 0$ and if the curvatures and the diameters are uniformly bounded for all times, then for every sequence $t_i \to \infty$ there exists a subsequence so that $g(t_i + t) \to h(t)$ as $i \to \infty$, where $h(t)$ is a Ricci soliton. It turns out that if one of the limit solitons is integrable then the limit is unique up to diffeomorphisms.

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Ricci flow of L^∞ metrics on three Manifolds

We consider the Ricci flow

$$\frac{\partial}{\partial t} g_{ij} = -2Ricci(g)_{ij},$$

of Riemannian metrics whose initial value $g_0 = g(0)$ is not necessarily smooth but which is controlled by a smooth background metric, in the sense that

$$\frac{1}{c}h \leq g_0 \leq ch,$$

for some smooth metric h. In particular we prove the following theorems.

Theorem 1. Let $(M^n, g(t))_{t\in[0,T)}$ be a smooth solution to the Ricci-flow, where $\frac{1}{c}h \leq g(\cdot,t) \leq ch$, for all $t \in [0,T)$. Then the solution may be extended to $(M, g(t)_{t\in[0,T+\epsilon)}$ for some small $\epsilon > 0$.

As an application we obtain the following theorem.

Theorem 2. Let $(M^3, {}^ig)$,$i \in \mathbb{N}$ be a family of smooth metrics which satisfy $\frac{1}{c}h \leq {}^ig \leq ch$, for some constant c independent of i, and $sec({}^ig) \geq -\epsilon(i)$ where $\epsilon(i) \to 0$ as $i \to \infty$. Then there exists a smooth metric g' on M^3 such that $sec(g') \geq 0$ and so M^3 may be differentially/topologically classified using the theorem of R.Hamilton [Ha].

[Ha] Hamilton, R. *Four-manifolds with positive isotropic curvature*, Comm. Anal. Geom. 5 (1997), no. 1, pp. 1–92.

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Mean curvature flow with surgeries of 2-convex hypersurfaces

In this talk (joint work with G. Huisken) we introduce a surgery procedure for mean curvature flow, which allows us to continue the flow after the singular time for certain classes of surfaces. In each surgery we remove a cylindrical region with high curvature and replace it by two spherical caps. We then restart the flow until the next singularity occurs. We can prove that, after a finite number of surgeries, the remaining pieces are diffeomorphic to spheres or to tori, so that the topology of the initial manifold can be reconstructed. Our procedure applies to surfaces of dimension at least three which are 2-convex, i.e. the sum of the two smallest principal curvatures is positive everywhere. The proof relies on the analysis of singularities for mean curvature flow, and on some new a priori estimates valid for the flow of 2-convex hypersurfaces.

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A flow approach to Nirenberg's problem and to the problem of prescribed Q-curvature

We describe an alternative approach to the existence results of Chang-Yang for metrics of prescribed scalar curvature on S^2 via the prescribed curvature flow. Moreover, we give an example showing that the results of these authors in general cannot be improved.
The flow approach may be carried over to the analogous higher-dimensional evolution problem for prescribed Q-curvature on S^4, giving rise to existence results which improve the ones previously obtained by Brendle by other methods.

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Boundary behaviors of compact manifolds with nonnegative scalar curvature

Let M be a compact Riemannian three manifold with smooth boundary Σ. It is assumed that (i) M has nonnegative scalar curvature; (ii) Σ has positive Gauss curvature and (iii) Σ has positive mean curvature. We shall use the idea of quasi-spherical metrics introduced by Bartnik together with a generalized form of the positive mass theorem of Schoen-Yau and Witten to study the boundary behaviors of M. Our study is related to the Brown-York quasi-local mass in general relativity. Generalizations by relaxing (i), (ii) or (iii) mentioned above will also be discussed.

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New Applications of Mean Curvature Flow to Minimal Surface Theory

I will show how mean curvature flow can be used to partially answer some old questions about densities of minimal cones. The methods also give some new results about densities of singularities in mean curvature flow.

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