Modular invariants and NIM-reps

Matthias R Gaberdiel (ETH Zürich, Switzerland),
Terry Gannon (University of Alberta)

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During the time of this program we have continued to work on different problems regarding D-branes of WZW models. Let us begin by briefly sketching the context.

One of the best understood string theories are the Wess-Zumino-Witten (WZW) models that describe closed strings propagating on a target space that is a Lie group G. The algebraic structures governing these models are affine Kac-Moody algebras.

As has become clear in recent years, many closed string theories possess D-branes. A D-brane is simply a submanifold of the target space on which the end points of additional open string degrees of freedom can lie. Because of this open string point of view, D-branes can be analysed and described using conformal field theory techniques (although they are actually 'non-perturbative' objects from the closed string point of view).

D-branes are dynamical objects that can in particular decay in various ways. In order to understand their dynamics it is useful to determine the invariant charges that characterise different configurations of D-branes. It is believed that the corresponding charge groups agree with certain K-theory groups. For the WZW models, the relevant K-groups are specific twisted K-groups that have been worked out in [1, 2].

A lot of work has already been done on D-branes in WZW models. In particular, the D-branes that preserve the full affine symmetry (possibly up to an automorphism) have been constructed. For the case where the group manifold is simply connected their charges have also been determined [3, 4, 5, 6]. (A systematic analysis for the case of non-simply connected Lie groups was recently begun in [7], see also [8, 9].) While the resulting charge groups agree beautifully with the independent K-theory calculations, it has became clear that, apart for some small rank exceptions, the D-branes that preserve the full affine symmetry do not account for all the K-theory charges. [For example, for the case of $G = \mathrm{SU}(n)$ the charge group that is predicted by K-theory consists of 2^{n-2} copies of some finite cyclic group Z_d, while the above D-brane constructions can only account for two of these summands (for $n \geq 3$).] This raises the question of how to construct the D-branes that account for the remaining charges.

This is the problem we attacked during our time at BIRS.[1] [For simplicity, we restricted ourselves to the case of $\mathrm{SU}(n)$, although it should be straightforward to generalise our constructions to arbitrary (simply-connected) groups.] In particular, we managed to find two constructions that seem to generate the remaining D-brane charges. The first construction [10] is based on the suggestion of [4] that the remaining charges should be related by some sort of T-duality to the original untwisted and twisted branes. We have shown that there are precisely 2^{n-2} different constructions that can be obtained in this manner. While these boundary states break in general the affine symmetry, their open string spectra can still be described in terms of twisted representations of the affine symmetry

[1] The work was also done in collaboration with a student of one of us.

algebra. As a consequence their charges can be determined, and we could show that each of the 2^{n-2} different constructions leads to the same charge group Z_d.

While this construction is quite suggestive, the geometric interpretation of the different D-branes is not obvious. In a second paper [11] we therefore gave a different construction that was more geometrically motivated. It is well known that the group SU(n) is rationally homotopy equivalent to a product of odd dimensional spheres

$$\mathrm{SU}(n) \cong S^{2n-1} \times S^{2n-3} \times \cdots \times S^3 \,. \tag{1}$$

Each of these spheres comes from a coset space $S^{2m-1} \cong \mathrm{SU}(m)/\mathrm{SU}(m-1)$, and thus homotopically we can think of SU(n) as the product of these coset spaces.

From a conformal field theory point of view we can decompose the space of states in terms of these coset algebras, and we can then construct D-branes that preserve their product. In fact, we managed to find 2^{n-2} different classes of D-branes that have this property. This multiplicity arises because for each factor of (1) (except the S^3 factor), there are two possible constructions that seem to describe D-branes whose world-volume does or does not wrap the corresponding sphere. Given the close connection of K-theory to homology [4], this then suggests that these D-branes generate in fact the full K-group.

Technically, the two constructions [10] and [11] are very similar indeed. The analysis of the charges is more unambiguous for the first construction, while the geometric interpretation is more transparent for the second. Taken together, they therefore give strong support to the assertion that either of them describes a collection of D-branes whose charges generate the full K-group.

Our two weeks at BIRS were again very productive indeed. We found the environment beautifully conducive to research. We also found Andrea Lundquist very helpful. We hope to be able to visit again some time in the future!

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