ABSTRACTS: LOCALIZED PATTERN FORMATION

Abstract: Rustum Choksi (Simon Fraser)
Title: Pattern Formation in two Physical Systems: Diblock Copolymer Melts and the Intermediate State of a Type-I Superconductor

I will present two physical problems in which pattern formation can be modeled via the minimization of a certain (nonlocal) free energy. In each case, I will briefly discuss the origin and derivation of the free energy. I will then turn to questions and results pertaining to scales and patterns for minimizers in several space dimensions.

Abstract: Chris Cosner (U. Miami)
Title: Aggregation and its Effects on Population Dynamics

Classical models for population dynamics with spatial dispersal typically assume that dispersal occurs by passive diffusion, perhaps with advection. Some models from the ecological literature propose nonlinear diffusion as a mechanism leading to aggregation. Replacing passive diffusion with nonlinear diffusion of the type proposed to model aggregation can lead to changes in the possible dynamics supported by the models. In logistic models it can create an Allee effect, where small populations go extinct but populations above some threshold persist. Stronger versions of nonlinear diffusion can lead to ill-posed problems. In this talk I will describe some nonlinear diffusion models in population dynamics, a few results, and some topics for further research on such phenomena. The analytic results are based on bifurcation theory and classical methods in partial differential equations.

Abstract: E. N. Dancer (U. Sydney)
Title: Stable and Finite Morse Index Solutions with Small Diffusion

We relate these problems in 2 or 3 dimensions to problems on the whole of space. We show in some cases that we obtain simple answers which are independent of domain shape (which is seldom true in other cases).

Abstract: Paul C. Fife (U. Utah)
Title: A Nonregular Grain Boundary Profile in a Material with Two Order Parameters

A 3-order-parameter phase field model for a material undergoing an order-disorder transition was introduced by Braun, Cahn, McFadden and Wheeler, and related models were studied after that. The mathematical structure of phase interface profiles for the original model was and is the subject of studies by Alikakos, Bates, Cahn, Fife, Fusco, and Tanoglu. This report is part of the latter effort. The physical problem is to determine grain boundary profiles separating a disordered from an ordered region, ordering being well defined in terms of crystalline structure. Mathematically, the profiles are represented by heteroclinic solutions of the phase field equations in 4-dimensional phase space. When the material has large anisotropy, the mathematical problem may involve the perturbation of a singular heteroclinic from 2-space to 4-space. The existence and properties of such a perturbed profile are proved. The singular limit (unperturbed) profile is known, continuous, but not differentiable. This last property is the source of the major difficulty.

Abstract: Thomas Hillen (U. Alberta)
Title: Volume-Filling and Quorum-Sensing in Models for Chemosensitive Movement

Some of the continuum models for chemotaxis are unstable in the sense that they can lead to finite time blow-up, or "overcrowding" scenarios (see the survey-talk of Horstmann). Cell overcrowding occurs since the classical models ignore the finite size of individual cells and the behaviour of cells at higher densities. With Painter, we extended the classical chemotaxis models to incorporate density dependence that precludes blow-up. In this talk, I present a number of approaches by which such equations can arise based on biologically realistic mechanisms, including the finite size of individual cells - "volume filling", the employment of cell density sensing mechanisms - "quorum-sensing", and finite sampling radius of the cells. We show the existence of nontrivial steady states, we prove global existnece of solutions and we study the traveling wave problem. A comprehensive numerical exploration of the model reveals a wide variety of interesting pattern forming properties, and we present the underlying bifurctaion structure.
(with Painter (Edinburgh) and Potapov (Alberta)).

Abstract: Dirk Horstmann (U. Koeln)
Title: Comparison of a Forward-Backward Parabolic Equation in Population Dynamics with its Time-Delay Approximations

Movement is a fundamental process for all biological organisms, ranging from single cell level to the population level. The movement of several species can be modeled by two equations. Such model can be given for example by one parabolic partial differential equation

$$p_t = \Delta(T(w)p)$$

for the species and by one ordinary differential equation

$$\epsilon w_t = F(p, w)$$

for a "control substance" that influences the movement of the species. In some cases those models reduce to a single parabolic partial differential equation which might lead to an ill-posed mathematical problem. However the two equation models with the PDE and the ODE can be viewed as an approximation of the single equation that takes time delays into account. In this talk I present the results from my joined works with H. G. Othmer and K. J. Painter where we compared for one concrete model the asymptotic behaviour of the solution of the non-local in time or time delay problem with the asymptotic behaviour of the solution of the corresponding single equation problem, which is a forward-backward problem in this specific case.

Abstract: Dirk Horstmann (U. Koeln)
Title (Survey Talk): From 1970 Until Present: The Keller-Segel Model in Chemotaxis and its Consequences

This talk summarizes various aspects and results for some general formulations of the classical chemotaxis models also known as Keller-Segel models. It will survey the results for the most common formulation of the classical model for positive chemotaxis and will focus on topics like steady state analysis, finite and infinite time blow-up of the solution and the existence of traveling waves solutions for some chemotaxis models.

Abstract: Tasso Kaper (Boston U.)
Title: Semi-Strong Pulse Interactions in some Reaction-Diffusion Systems

Pulse-pulse interactions play central roles in a variety of pattern formation phenomena, including self-replication. In this talk, we present theory for the semi-strong interaction of pulses in a class of singularly perturbed, coupled reaction-diffusion equations that includes the (generalized) Gierer-Meinhardt, Gray-Scott, Schnakenberg, and Thomas models, among others. Geometric conditions are determined on the reaction kinetics for whether the pulses in a two-pulse solution attract or repel, and ordinary differential equations are derived for the time-dependent separation distance between their centers and for their wave speeds. In addition, conditions for the existence of stationary two-pulse solutions are identified, and the interactions between stationary and dynamically-evolving two-pulse solutions are studied. Also, the stability of these solutions is studied using the NonLocal Eigenvalue Problem method developed jointly with A. Doelman (Amsterdam) and R. Gardner (Amherst). The theoretical results are illustrated on a series of examples. In two of these, which are related to the classical Gierer-Meinhardt equation, we find that the pulse amplitudes blow up in finite time. Moreover, the blowup of stationary one-pulse solutions and of dynamically-varying two-pulse solutions occurs precisely at the parameter values for which the theory we develop predicts that these solutions should cease to exist. Finally, for one of these examples, we discover a new type of codimension two point in which the bifurcation curve of pulse-splitting and the bifurcation curve of blowup intersect.

Abstract: Theodore Kolokolnikov (UBC)
Title: The Coupled Regime of the Gray-Scott Model in 1-D

Most of the recent work on the Gray-Scott model concerns the parameter regimes where one of the chemicals diffuses much slower than the other. This ”weak coupling” assumption makes it possible to obtain full analytical results. In this work we look at the regime of the Gray-Scott model where the two chemicals are strongly coupled inside the core of the spike, but weakly coupled outside the core of the spike. We use a hybrid numerical/asymptotic method to make predictions about the number of spikes on an interval of a given length. We also analyse the stability of a spike solution with respect to travelling wave instabilities. We obtain thresholds for existence of the travelling wave in terms of the parameters of the problem and the domain length.

Abstract: Rachel Kuske (UBC)
Title: Modulation Equations for Localized Patterns Away ¿From Criticality

Modulation equations describe the behavior of complex systems over long scales. However, their validity is often limited to near-criticality. A new multi-scale approach, combining energy arguments and balance of nonlinearities, yields modulation equations for localized buckling of a strut away from the critical load, where standard asymptotics and normal forms fail. Immediate connections to heterogeneous patterns in other applications are shown. The approach is illustrated via simple one-dimensional models, motivated by numerics and experiments.

Abstract: Bernard Matkowsky (Northwestern)
Title: Dynamics of Hot Spots in Solid Fuel Combustion

We consider the gasless combustion model of the SHS (Self-Propagating High Temperature Synthesis) process in which combustion waves are employed to synthesize desired materials. Specifically, we consider the combustion of a solid sample in which combustion occurs on the surface of a cylinder of radius R. We consider solution behavior as R is increased. This parameter is important for technological applications, as it is often desirable to synthesize large samples of the desired product. For the fixed value of the Zeldovich number considered, if R is sufficiently small, slowly propagating planar pulsating flames are the only modes observed. As R is increased transitions to more complex modes of combustion occur, including (i) traveling waves (TWs), i.e., spin modes in which one or several symmetrically spaced hot spots (localized temperature maxima) rotate around the cylinder as the flame propagates along the cylindrical axis, thus following a helical path, (ii) counterpropagating (CP) modes, in which spots propagate in opposite angular directions around the cylinder, executing various types of dynamics, (iii) alternating spin CP modes (ASCP), where rotation of a spot around the cylinder is interrupted by periodic events in which a new spot is spontaneously created ahead of the rotating spot. The new spot splits into counterpropagating daughter spots, one of which collides with the original spot leading to their eventual mutual annihilation, while the other continues to spin, (iv) modulated traveling waves (MTWs) consisting of either one or two symmetrically located rotating spots which exhibit a periodic modulation in speed and temperature as they rotate (v) asymmetric traveling waves (ATWs) in which two spots of unequal strength and not separated by angle π, rotate together as a bound state, (vi) modulated asymmetric traveling waves (MATWs) in which the two asymmetric spots oscillate in a periodic manner as they rotate, alternately approaching each other and then moving apart periodically in time, (vii) asymmetric ASCP modes (AASCP) in which a slowly varying bound state of two spots rotates around the cylinder with the leading spot, and subsequently the trailing spot, exhibiting episodes of ASCP behavior, and others.

Abstract: Yasumasa Nishiura (Hokkaido U.)
Title: Scattering in Dissipative Systems

Scattering of particle-like patterns in dissipative systems is studied, especially we focus on the issue how the input-output relation is controlled at a head-on collision where traveling pulses or spots interact strongly. It remains an open problem due to the large deformation of patterns at a colliding point. We found that special type of unstable steady or time-periodic solutions called scattors and their stable and unstable manifolds direct the traffic flow of orbits. Such scattors are in general highly unstable even in 1D case which causes a variety of input-output relations through the scattering process. We illustrate the ubiquity of scattors by using the complex Ginzburg-Landau equation, the Gray-Scott model and a three-component reaction diffusion model arising in gas-discharge phenomena.

Abstract: Yasumasa Nishiura (Hokkaido U.)
Title (Survey Talk): Toward the Understanding of Strong Interactions Among Localized Patterns

There are two types of strong interactions (or instabilities) for particle-like patterns, namely intrinsic and extrinsic ones. Self-replication and self-destruction are the typical examples of intrinsic type. Strong collision and the resulting scattering between traveling spots is of extrinsic one. Conventional perturbative methods don't work in this regime and there remain lots of open problems due to the large deformation of patterns not only in mathematical sense but also physical or even computational sense. I will present several potentially useful vewpoints and tools to explore this fertile ground.

Abstract: Hans Othmer (U. Minnesota)
Title: From Individual to Collective Behavior in Bacterial Chemotaxis

Bacterial chemotaxis is widely studied from both the microscopic (cell) and macroscopic (population) points of view, and in this talk we connect these widely-separated levels of description by deriving the classical macroscopic description for chemotaxis from a microscopic model of the behavior of individual cells. The analysis is based on the velocity jump process for describing the motion of individuals, wherein each individual carries an internal state that evolves according to a system of ordinary differential equations forced by a time- and/or space-dependent external signal. We derive a macroscopic system of hyperbolic differential equations from this velocity jump process using moment closure techniques. We also reduce this macroscopic system to a single second order hyperbolic equation which, in a suitable limit, reduces to a classical chemotaxis equation in which the chemotactic sensitivity is now a known function of parameters of the internal dynamics.

Abstract: John E. Pearson (Los Alamos)
Title: Sheet Excitability and Nonlinear Wave Propagation

In the *Xenopus laevis* oocyte, calcium ion channels are clustered in a thin shell near the outer cell wall. Motivated by this morphology we study the effect of "sheet excitability" in an idealized reaction–diffusion system with a 2-dimensional sheet of sources embedded in 3-dimensional space. We find waves that undergo propagation failure with *increasing* diffusion coefficient and a scaling regime in which the wave speed is independent of the diffusion coefficient.

Abstract: Xiaofeng Ren (Utah State U.)
Title: Lamellar and Wriggled Lamellar Solutions of the Diblock Copolymer Problem

The type A and type B monomers in a diblock copolymer system often form A-rich and B-rich microdomains. On a larger scale these phase domains give rise to a morphological pattern. We study the widely observed lamellar and wriggled lamellar patterns using the Ohta-Kawasaki model in which the free energy density field depends nonlocally on the monomer composition field. In one dimension the problem is reduced by the Gamma-convergence technique to a finite dimensional minimization problem. For each K there exists a 1-D local minimizer with K+1 microdomains and K domain walls. Among these 1-D local minimizers there is the 1-D global minimizer that has optimal spacing between the domain walls. These 1-D local minimizers are extended trivially to two dimensions to give solutions of lamellar patterns to the Euler-Lagrange equation. The stability of these solutions are studied and their spectra are found. A 1-D local minimizer is stable in 2-D only if it has sufficiently many domain walls. The 1-D global minimizer is near the borderline of 2-D stability. This interesting phenomenon is related to the existence of wriggled lamellar solutions as seen from the bifurcation theory. The stability of the wriggled lamellar solutions are determined in the theory after some careful calculations.

Abstract: Xiaofeng Ren (Utah State U.)
Title (Survey Talk): The Chiral Liquid Crystal Problem

A chiral liquid crystal system is modelled by two fields: the scalar molecular chirality field and the vector molecular director field of unit length. The free energy of the system contains three parts: the Ginzburg-Landau free energy that depends on the chirality field, the nematic liquid crystal free energy that depends on the director field, and a coupling energy that relates the two fields. A particular pattern in the system of soliton-stripe type is discussed.

Abstract: Angela Stevens (Max-Planck)
Title: Kinetic Models for Chemosensitive Movement and their Macroscopic Limits

A widespread phenomenon in moving microorganisms and cells is their ability to orient themselves in dependence of chemical signals. This often leads to specific pattern forming processes or aggregation of cells.

Kinetic models for chemosensitive movement are discussed which take into account a variety of evaluations of the chemical stimulus which subsequently influence the motion of the respective species on the macroscopic level.

The basic type of model was discussed, among others, by Alt; Othmer, Dunbar and Alt; and by Othmer and Hillen. Chalub, Markowich, Perthame and Schmeiser rigorously proved that in three dimensions, these kind of kinetic models lead to the classical Keller-Segel model as its drift-diffusion limit, when the equation for the chemical signal is of elliptic type and local and non-local evaluations of the density of the chemical signal by the chemosensitive species are taken into account.

In this talk the rigorous derivation of Keller-Segel type systems and their variants is given in case the equation for the chemical signal is of either parabolic or elliptic type (also in 2 dimensions) and for situations where additionally evaluations of gradient fields of the chemical signal can be taken into account. Under suitable structure conditions existence of global solutions for the kinetic model can be shown, also in situations where the Keller-Segel model shows finite time blowup. (Joint work with Hyung Ju Hwang and Kyungkeun Kang)

Abstract: Michael J. Ward (UBC)
Title: Oscillatory and Competition Instabilities of Localized Patterns in the Gierer-Meinhardt and Gray-Scott Models

Novel dynamical behaviors that occur for spike-type solutions to the one-dimensional Gierer-Meinhardt reaction-diffusion system and the Gray-Scott model in the low feed-rate regime are highlighted. There are parameter regimes where spikes exhibit synchronous oscillations in their amplitudes due to a hopf bifurcation, and other regimes where the spikes exhibit competition instabilities leading to the destruction of certain spikes in a spike sequence. A rigorous and asymptotic analysis of these two different types of instabilties is performed. The analysis is based largely on the spectral properties of a certain nonlocal eigenvalue operator. Numerical computations are performed that validate the asymptotic theory. (joint work with Juncheng Wei (CUHK) and Theo Kolokolnikov (UBC)).

Abstract: Juncheng Wei (Chinese U. Hong Kong)
Title: On Two-Dimensional Gray-Scott Model: Symmetric and Asymmetric spots and rings

We discuss recent progress towards the following Gray-Scott model:

$$v_t = \epsilon^2 \Delta v - v + Auv^2$$

$$\tau u_t = D\Delta u + 1 - u_u v^2$$

in a bounded two-dimensional domain. There are regimes of A: (I) $A = O(\epsilon)$ (II) $A = O(\epsilon^{\frac{1}{2}})$ (III) $A = O(1)$. In regime I, we show the existence of both symmetric and asymmetric spotty patterns. The stability of these spotty patterns are analyzed in terms of critical thresholds for D. In regime II, we show the existence of ring-like solutions in a radially symmetric domain. These solutions are shown to be unstable with some node m. In regime III, we show the existence of ring-like solutions which exhibit zig-zig instabilities for some mode m. Self-replicating rings and rings-breaking-into-spots are observed and analyzed. (Joint work with Theo Kolokolnikov and M. Winter)

Abstract: Juncheng Wei (Chinese U. Hong Kong)
Title (Survey Talk): Some Open Problems in the Gierer-Meinhardt System

The following Gierer-Meinhardt system

$$A_t = \epsilon^2 \Delta A - A + \frac{A^p}{H^q}, \quad x \in \Omega, \quad t > 0$$

$$\tau H_t = D\Delta H - H + \frac{A^r}{H^s}, \quad x \in \Omega, \quad t > 0$$

$$\frac{\partial A}{\partial \nu} = \frac{\partial H}{\partial \nu} = 0 \text{ on } \partial\Omega$$

has been a subject of study over recent years. In this talk, I will give an update on the progress made and present some open problems.

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