THE BIRS FIVE DAYS WORKSHOP

CALABI–YAU VARIETIES AND MIRROR SYMMETRY

DECEMBER 6–11, 2003

WORKSHOP SCHEDULES (TENTATIVE)

ABSTRACTS

DECEMBER 7, 2003

9:15am–10:15am: Jim Bryan (University of British Columbia)

Topological Quantum Field Theory and the Gromov–Witten Theory of Curves in Calabi–Yau Threefolds

Topological Quantum Field Theory, as formulated by Atiyah, has provided a general frame-work for understanding invariants of manifolds. The structure of TQFTs in dimension $1+1$ (i.e. surfaces with boundaries) is completely understood by elementary means yet they can still yield surprising results. For each positive integer d, we define a one-parameter family of $(1+1)$-dimensional TQFTs $Z_d(t)$ which specializes at $t=0$ to the famous Witten-Dijgraaf-Freed-Quinn TQFT for gauge theory with finite gauge group S_d (the d-th symmetric group). Our family of TQFTs completely encodes all the degree d local Gromov-Witten invariants of a curve (of arbitrary genus) in a Calabi-Yau threefold. This provides us with a "structure theorem" for these local invariants (a.k.a. multiple cover formulas). Using these ideas we completely determine the local invariants for $d < 7$.

10:45am–11:45am: Albrecht Klemm (University of Wisconsin, Physics)

The Topological Vertex

Authors: Mina Aganagic, Albrecht Klemm, Marcos Marino, Cumrun Vafa

We construct a cubic field theory which provides all genus amplitudes of the topological A-model for all non-compact Calabi-Yau toric threefolds. The topology of a given Feynman diagram encodes the topology of a fixed Calabi-Yau, with Schwinger parameters playing the role of Kahler classes of Calabi-Yau. We interpret this result as an operatorial computation of the amplitudes in the B-model mirror which is the Kodaira-Spencer quantum theory. The only degree of freedom of this theory is an unconventional chiral scalar on a Riemann surface. In this setup we identify the B-branes on the mirror Riemann surface as fermions related to the chiral boson by bosonization.

2:00pm–3:00pm: Xi Chen and James D. Lewis (University of Alberta)

Hodge D-Conjecture for $K3$ and Abelian Surfaces

We prove that the real regulator map is surjective on a general K3 or Abelian surface. The heart of the proof involves the use of rational curves on K3 and a degeneration argument. It is closely related to the recent progress on the enumerative geometry on K3. This is joint work with James Lewis.

3:30pm–4:30pm: Chuck Doran (Columbia University)

Integral Structures, Toric Geometry, and Homological Mirror Symmetry

We establish the isomorphisms over $\mathbf{Z}$ of cohomology/K-theory, global monodromy, and invariant symplectic forms predicted by Kontsevich's Homological Mirror Symmetry Conjecture for certain one dimensional families of Calabi-Yau threefolds with $h^{2,1}=1$. These families arise as hypersurfaces or complete intersections in Gorenstein toric Fano varieties, and their mirrors are described by the Batyrev-Borisov construction. Our method involves (1) classifying all rank four integral variations of Hodge structure over $\mathbf{P}^1 \setminus \{0,1,\infty\}$

with maximal unipotent local monodromy about 0 and local monodromy about 1 unipotent of rank 1, and (2) checking, using properties of nef partitions of reflexive polytopes, that the **Z**-VHS of our Calabi-Yau families match those picked out by the K-theory of their mirrors via the HMS Conjecture. This is joint work with John Morgan.

4:45pm–5:45pm: Andrey Todorov (University of California Santa Cruz)

Higher Dimensional Analogues of Dedekind Eta Function

It is a well known fact that the Kronecker limit formula gives an explicit formula for regularized determinants of flat metrics on elliptic curves. It established the relation between the regularized determinant of the flat metrics on elliptic curves and their discriminants. This relation can be interpreted as follows; There exists a holomorphic section (multivalued) of the dual of the determinant line bundle such that its L^2 norm is equal to the regularized determinant of the Laplacian acting on $(0, 1)$ forms. Kronocker limit formula established that the holomorphic section constructed from the determinant line bundle is the Dedekind eta function.

In the talk we discuss the existence of the analogue of the Dedekind eta function for K3 surfaces and CY manifolds. The construction of the generalized Dedekind eta function is based on the variational formulas for the determinants of the Laplacians of a Calabi-Yau metric acting on functions and forms of type $(0, 1)$ on CY manifolds, K3 surfaces and Enriques surfaces. Based on the variational formulas we will establish the existence of a holomorphic section of some power N of the dual of determinant line bundle on the moduli space of odd dimensional CY manifolds whose L^2 norm is the N^{th} power of the regularized determinant of the Laplacian acting on $(0, 1)$. This holomorphic section of the determinant line bundle is the analogue of the Dedekind eta function for odd dimensional CY manifolds. In case of even dimensional CY manifold and we will show the existence of a holomorphic section of the relative dualizing sheaf of the moduli space.

In case of K3 surfaces the construction of the Dedekind Eta function is done on the moduli of Kähler-Einstein-Calabi-Yau metrics and then projected to the moduli of polarized algebraic K3 surfaces.

We will discuss also that the L^2 norm on the relative dualizing sheaf is a good metric in the sense of Mumford. This implies that the Weil-Petersson volumes of the moduli spaces of CY manifolds are rational numbers. When M is a CY threefold we will outline how to prove that the regularized determinant of the Laplacian acting on $(0, 1)$ forms is bounded and that the section η^N vanishes on the discriminant locus.

DECEMBER 8, 2003

9:15am–10:15am: Brian Forbes (University of California Los Angeles)

Open String Mirror Maps from Picard–Fuchs Equations on Relative Cohomology

A method for computing the open string mirror map and superpotential, using an extended set of Picard- Fuchs equations, is presented. This is based on techniques used by Lerche, Mayr and Warner. For X a toric hypersurface and Y a hypersurface in X, the mirror map and superpotential are written down explicitly. As an example, the case of K_{p^2} is worked out and shown to agree with the literature.

10:45am–11:45am: Eckart Viehweg (University of Essen)

Complex Multiplication, Griffiths–Yukawa Couplings, and Rigidity for Families of Hypersurfaces

Reporting on joint work with Kang Zuo: math.AG/0307398.

Let $M(d, n)$ be the moduli stack of hypersurfaces of degree $d > n$ in the complex projective n-space, and let $M(d, n; 1)$ be the sub-stack, parameterizing hypersurfaces obtained as a d-fold cyclic covering of the projective $n - 1$-space, ramified over a hypersurface of degree d. Iterating this construction, one obtains $M(d, n; r)$. The substack $M(d, n; 1)$ is rigid in $M(d, n)$, although the Griffiths-Yukawa coupling degenerates for $d < 2n$, hence in particular for Calabi-Yau hypersurfaces. On the other hand, for all $d > n$ the sub-stack

$M(d,n;2)$ deforms. One can calculate the exact length of the Griffiths-Yukawa coupling over $M(d,n;r)$. As a byproduct one finds a rigid family of quintic hypersurfaces over some 4-dimensional subvariety M of the moduli stack, and a dense set of points in M, where the fibres have complex multiplication.

2:00pm–3:00pm: Yi Zhang (Zhejiang University, China)

Some Results on Families of Calabi–Yau Varieties

The aim of the lecture is to show some new results related to the families of projective Calabi-Yau manifolds.

First, the author introduces concisely the results of rigid problem related to Shafarevich conjecture of Calabi-Yau which are included in the author preprint " The Rigidity of Families of Projective Calabi-Yau manifolds, Math.AG/0308034", i.e. he shows that some important families of Calabi-Yau manifolds are rigid,for examples:

(I) Lefschetz pencils of odd dimensional Calabi-Yau manifolds are rigid;

(II) Strong degenerate families (in some sense, an not need to be CY manifolds) are rigid;

(III) Families of CY manifolds admitting a degeneration with maximal unipotent monodromy must be rigid.

The main methods of the author to attack the problems is that the degenerate theory of variation of Hodge Structure, Yang-Mills theory of Higgs bundle and Deligne-Katz's theory on monodromy,etc.

Initiated by the results of rigidity problems,the author study the important and interesting object in family geometry: Mumford-Tate group. The author wants to understand how Mumford-Tate groups control the families,especially the families of Calabi-Yau threefolds? Are there necessary relations between Mumford-Tate groups and rigidity problems? For example,he shows some relations between the global monodromy group and Mumford-Tate group.

3:30pm–4:30pm: Rolf Schimmrigk (Kennesaw State University)

Complex Multiplication of Calabi–Yau Varieties and String Theory

Abelian varieties with complex multiplication can be identified as the basic cohomological building blocks of certain types of Calabi-Yau manifolds. It is therefore possible to define the notion of complex multiplication for Calabi-Yau spaces via the complex multiplication type of these abelian varieties. The aim of this talk is to show how this symmetry illuminates the exactly solvable conformal field theoretic nature of Calabi-Yau varieties.

4:45pm–5:45pm: Wei-Dong Ruan (University of Wisconsin)

Generalized Special Lagrangian Torus Fibration for Calabi–Yau Manifolds

In light of SYZ conjecture, special Lagrangian torus fibration play important role in mirror symmetry. In this talk, we will discuss new examples of special Lagrangian submanifolds and the construction of global generalized special Lagrangian torus fibration for Calabi-Yau manifolds.

8:00pm–9:00pm: Kentaro Hori (Physics, University of Toronto)

Calabi–Yau Orientifolds

I introduce orientifolds to mathematicians and discuss some applications. Orientifolds are associated with unoriented strings. Real algebraic geometry plays an important role, just as symplectic geometry and algebraic geometry did for oriented strings.

DECEMBER 9, 2003

9:15am–10:15am Matt Kerr (University of California Los Angeles)

Constructing Nontrivial Cycles on Abelian and Calabi-Yau Varieties

We collect together some techniques and results due to A. Collino, J. Lewis, S. Muller-Stach, S. Saito and others; the main object of study is the indecomposable part of $CH^p(X,n)$ in the cases $3 \geq p \geq n \geq 0$, where X is a Calabi-Yau or Abelian 3-fold or surface. Our aim is to discuss regulator formulas (including those developed in our work), connectivity results, degeneration techniques, and differential equations satisfied by regulator "periods" in families. We will also indicate some interesting open problems.

10:45am–11:45am: Shi-shyr Roan (Academia Sinica, Taipei, Taiwan)

Rational Curves in Rigid Calabi–Yau Threefold

We determine all the Kummer-surface-type Calabi-Yau (CY) 3-folds, i.e., those $\widehat{T/G}$ obtained by resolution of a 3-torus-orbifold T/G with only isolated singularities. There are only two such CY spaces: one with $G = Z_3$, and the other with $G = Z_7$. These CY 3-folds $\widehat{T/G}$ are all rigid, hence no complex structure deformation for the varieties. We further investigate problems of rational curves in $\widehat{T/G}$ not contained in exceptional divisors, by considering the counting number d of points in a rational curve C meeting exceptional divisors in a certain manner. We have obtained the constraint on d. With the smallest number d, the complete solution of C in $\widehat{T/G}$ is obtained for both cases. In the case $G = Z_3$, we have derived an effective method of constructing C in $\widehat{T/G}$, and obtained the explicit forms of rational curves for some other d by the method.

DECEMBER 10, 2003

9:15am–10:15am: Belazs Szendroi (University of Utrecht)

Calabi–Yau Threefolds in Weighted Homogeneous Varieties

Reprting on joint work with Anita Buckley.

Let (X, D) be a Calabi-Yau threefold with quotient singularities, polarized by an ample Q-Cartier divisor. We prove a formula expressing the dimension of the vector space $H^0(X, nD)$ in terms of global numerical invariants of (X, D) and local invariants of D at the quotient singularities of X. Based on this formula, we construct several new families of Calabi-Yau threefolds in weighted homogeneous varieties, generalizations of weighted projective spaces introduced by Corti and Reid. In some cases, we show how to compute Hodge numbers of (smooth Calabi-Yau models of) these threefolds using birational geometry.

10:45am–11:45am: Shinobu Hosono (University of Tokyo)

GKZ Hypergeometric Series, Mirror Symmetry, and Singularity Theory

In the last workshop at Fields Institute(July, 2001), I talked about GKZ hypergeometric series taking values in the cohomology group of a Calabi-Yau manifold, and made a conjecture on the period integrals of the mirror Calabi-Yau manifold. In the case of two dimensional toric (non-compact) Calabi-Yau manifolds, I will verify the conjecture by relating the hypergeometric series to the integral solutions of K. Saito's differential equation in singularity theory. I will also present some three dimensional examples, and try to refine the conjecture.

2:00pm–3:00pm: Marie José Bertin (Université Pierre et Marie Curie (Paris 6))

Mahler's Measure and L-Series of $K3$ Hypersurfaces

We express in terms of Eisenstein-Kronecker series the Mahler's measure of two families of polynomials defining $K3$ hypersurfaces. For some of these polynomials we relate their Mahler's measure with the L-series of the corresponding $K3$-surface.

3:30pm–4:30pm: Klaus Hulek (University of Hannover)

Examples of Non-Rigid Modular Calabi–Yau Manifolds

Reporting on joint work with Helena Verrill.

In this talk we want to present some examples of non-rigid Calabi-Yau varieties whose L-series is modular. These examples are constructed by considering nodal Calabi-Yau varieties in the toric variety associated to the A_4 root lattice.

4:45pm–5:45pm: Kenichiro Kimura (University of Tsukuba)

K_1 of a self-product of a curve

Beilinson's conjectures on special values of L-functions predicts the existence of interesting higher Chow cycles on varieties over number fields. I will explain about the attempts to create such cycles mainly in the case of a self- product of a curve.

DECEMBER 11, 2003

9:15am–10:15am: Keiji Oguiso (University of Tokyo)

Simple Groups, Solvable Groups and $K3$ Surfaces

We characterize the following three particular K3 surfaces, among all the complex K3 surfaces, by means of finite group symmetries:

(1) the Fermat quartic K3 surface

$$x_1^4 + x_2^4 + x_3^4 + x_4^4 = 0\,.$$

(2) the Klein-Mukai quartic K3 surface

$$x_1^3 x_2 + x_2^3 x_3 + x_3^3 x_1 + x_4^4 = 0\,,$$

i.e. the cyclic covering of degree 4 of projective plane branched along the Klein quartic curve. This is a joint work with D.-Q. Zhang.

(3) the minimal resolution of

$$s^2(x^3 + y^3 + z^3) - 3(s^2 + t^2)xyz = 0$$

in $\mathbf{P}^1 \times \mathbf{P}^2$, i.e. the minimal resolution of the double cover, branched along two singular fibres, of the (rational) elliptic modular surface with level 3 structure. This is a joint work with J.H. Keum and D.-Q. Zhang.

These three are all singular K3 surfaces in the sense of Shioda. The surface (1) is uniquely characterized as the K3 surface admitting either the solvable finite group action of maximum order or the nilpotent finite group action of maximum order. The surface (2) (resp. (3)) is uniquely characterized as the K3 surface admitting an action of the maximal possible finite extension of the simple group $L_2(7)$ (resp. $L_2(9) \simeq A_6$). In each case, we also show the uniqueness of the groups and their actions.

By a result of Mukai, the finite simple (non-commutaive) groups which can act on some K3 surfaces are only $L_2(7)$, $L_2(9)$ and A_5, the first three groups in ATLAS. Among these three, the first two are the maximal simple groups (with respect to the inclusion of groups) which can acts on K3 surfaces. If possible, I would like to discuss about non-maximal A_5 case, too.

10:45am–11:45am : John McKay (Concordia University)

TBA, or Three Sporadic Groups and AFfine Lie Data

I promote two conjectures - one I discovered 25 years ago - and the other just this year. A deep connection exists between affine $E6, E7$, and $E8$ data, and certain Fischer involutions of $F24', B$, and M.

The groups of 27 lines on a 3-ic, and 28 bitangents on a 4-ic have a large significant literature but the 120 tritangent planes on a 6-ic curve of genus 4 do not. The fundamental groups of type $E6, E7, E8$ are the Schur multipliers of the corresponding sporadic groups. The second conjecture is the appearance of the class number, 194, of M as a Picard number in "Numerical Oddities" of hep-th/0002012 by Aspinwall, Katz and Morrison.