Self-Selection and Discrimination in Credit Markets

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Supplemental Proofs

Proof of Result 1: We show that $\pi(\theta \mid s)$ satisfies the monotone likelihood ratio property, which implies that $q'(s)$ must be positive, since $q(s)$ is the expectation of θ under π.[1]

$$\begin{aligned}\frac{\frac{\partial \pi(\theta \mid s)}{\partial s}}{\pi(\theta \mid s)} &= \frac{\frac{\partial p(s \mid \theta)}{\partial s} g(\theta)\omega(s) - p(s \mid \theta) g(\theta)\omega'(s)}{p(s \mid \theta) g(\theta)\omega(s)} \\ &= \frac{\frac{\partial p(s \mid \theta)}{\partial s}}{p(s \mid \theta)} - \frac{\omega'(s)}{\omega(s)}.\end{aligned} \tag{1}$$

Hence,

$$\begin{aligned}\frac{\partial}{\partial \theta} \frac{\frac{\partial \pi(\theta \mid s)}{\partial s}}{\pi(\theta \mid s)} &= \frac{\frac{\partial^2 p(s \mid \theta)}{\partial s \partial \theta} p(s \mid \theta) - \frac{\partial p(s \mid \theta)}{\partial s} \frac{\partial p(s \mid \theta)}{\partial \theta}}{[p(s \mid \theta)]^2} \\ &= \frac{\partial}{\partial s} \frac{\frac{\partial p(s \mid \theta)}{\partial \theta}}{p(s \mid \theta)},\end{aligned} \tag{2}$$

which is positive since p satisfies the monotone likelihood ratio property. It is worth noting that this proof holds regardless of the distribution of credit risk in the applicant pool. ♠

Proof of Result 2: Immediate from the monotonicity of $q(s)$. ♠

Proof that the probability of approval is increasing in applicant type: By the symmetry of p (part 2 of A1), $\alpha(\theta)$ can be rewritten as $\alpha(\theta) = \int_{-\infty}^{\theta} p(t \mid s^*) dt$. Differentiating with respect to θ gives $\alpha'(\theta) = p(\theta \mid s^*) > 0$. ♠

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[1] This follows because the monotone likelihood ratio property implies first-order stochastic dominance; see Milgrom (1981).