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Alberta-Montana Combinatorics and Algorithms Days

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1 Overview

The first weekend in June 2022 saw the inaugural *Alberta-Montana Combinatorics and Algorithms Days* hosted at the Banff International Research Station (BIRS). One purpose of the event was to bring together faculty and students from three Alberta universities (in Calgary, Lethbridge, and Edmonton) and the University of Montana (Missoula).

Combinatorics is the branch of mathematics concerned with finite sets: their properties, structures, and number. Studying the classic Rubik's Cube reveals the number of possible positions (it's $43,252,003,274,489,856,000$).

Understanding the cube's structure leads to efficient algorithms for solving it (an Algorithm being a sequence of well-defined instructions for solving a problem, answering a question, or even playing a game). In 2010, a group of researchers working with Google proved that every one of that staggering number of positions could be solved in no more than 20 moves.

The fields of Combinatorics and Algorithms became inextricably linked at the dawn of the computer age due to computers themselves being finite structures. This meeting was conceived to offer regional researchers in these fields an opportunity to share their recent successes, tackle open problems together, and expose their students to the latest methods and developments.

Demographics (briefly)

The workshop attracted participants from a broader geographical region than originally envisioned; to wit, there were speakers from Calgary, Edmonton, Lethbridge, Missoula, Winnipeg, and Toronto. Likewise, the 'vertical' representation went even deeper than the organizers had originally hoped. Of the participants, there were two undergraduate students (one of whom spoke), five graduate students (four of whom spoke), and two postdoctoral fellows ((both of whom spoke). Thus, the total of sixteen talks was rounded out by nine lectures given by junior and senior faculty.

2 Presentation highlights

Here follows a précis of each of the talks in their original chronological order.

Ting Han Wei (University of Alberta) opened the scientific program with a lovely introduction to the (highly combinatorial) game of Go. Starting from the basics, he brought the audience to the state-of-the-art for solving 'small' (i.e., up to 6×6) boards. Solving the full board (19×19) may be beyond the abilities of humans ever to solve.

Thomas Pender (University of Lethbridge) began his talk by grounding the gathering on elementary Hadamard matrix theory. He quickly worked toward modern generalizations including orthogonal designs; this included some attractive constructions of these objects.

In her talk, Anastasia Halfpap (University of Montana) presented strong new results on extremal combinatorics. These particularly showcased examples and theory in situations where two (or more) structures of completely different character can achieve extremality in Turán-type problems.

In the first invited (full-hour) talk, Mark Kayll (University of Montana) introduced a graph family generalizing the well-known Kőnig-Egerváry graphs. This lecture—based in part on [6]—considered both theoretical and algorithmic aspects of these 'Egerváry' graphs.

Kris Vasudevan (University of Calgary) walked his audience through deep connections between graph theory and neuroscience, particularly brain disorder studies.

The second invited lecture featured Joy Morris (University of Lethbridge), who discussed pursuit games on generalized Petersen graphs. Among many results, Dr. Morris presented joint work [8] with her 16-year old daughter Harmony Morris (thus pushing the vertical integration mentioned above to a level still deeper than earlier indicated).

Cory Palmer (University of Montana) used a Star Wars lens to introduce his talk on enumerating stable matchings in complete bipartite graphs $K_{n,n}$ (with linear preference rankings at each vertex). In perhaps the most striking scientific announcement of the workshop, Dr. Palmer walked us through his recent improvement (with Dömötör Pálvölgyi) from 131072^n to 3.55^n on the best-known upper bound for the maximum number of stable matchings; see [9].

The lecture by Bobby Miraftab (University of Lethbridge) introduced a new graph decomposition (due to Stavropoulos) generalizing tree-decompositions. These so-called 'median-decompositions' exhibit connections with chordality and hyperbolicity in graphs.

In the third invited lecture, Michael Cavers (University of Toronto, Scarborough) spoke on reconfiguring vertex colourings of graphs, a formal way to capture the notion of two colourings being incrementally close. This far-reaching concept has applications to the infamous 15-puzzle, to change-ringing (in bell choirs), and to the so-called Glauber dynamics Markov chain (among others). See [2] as an example of one of Dr. Cavers' many cited references.

Vlad Zaitsev (University of Lethbridge) presented several new results and proofs concerning optimal constant-weight ternary codes. His work (with Hadi Kharaghani and Sho Suda) generalizes a result first proved twenty years ago; see [7].

Closing out the main program for the day, Ramin Mousavi (University of Alberta) presented on a variation of the classical Steiner tree problem. His work (with Zachary Friggstad) addresses an important special case of the 'Directed Steiner Tree' problem, namely when the underlying graph is planar or, more generally, excludes a fixed minor.

After a break for dinner, Rob Craigen (University of Manitoba) gave a plenary talk sharing his career-long experience and insights on Mathematics Education in Canada. This was followed by a lively discussion fueled by all the resulting questions and comments on a topic so close to many of the hearts at the workshop.

Professor Craigen continued to hold the floor to open the second morning, when he spoke on circulant partial Hadamard matrices. Among other things, his presented work suggests a possible resolution of the 'circulant Hadamard matrix conjecture'.

In the workshop's penultimate invited lecture, Ryan Hayward (University of Alberta) gave a compelling introduction and overview of the board game Hex, on which he has published two recent books: [4] and [5]. Unlike Go and Chess, Hex offers a game that lends itself to proofs concerning strategy and even solvability.

The workshop's final invited lecture was delivered by Zachary Friggstad (University of Alberta). Entitled *Prize-Collecting Walks and Branchings in Directed Graphs*, his talk was exemplary in bringing together the workshop themes (combinatorics and algorithms). Here we saw combinatorics (from a digraph perspective), linear programming, and a combinatorial algorithm (to replace an LP with a prohibitive number of variables and constraints). As these ideas were also applied to the design of approximation algorithms for vehicle routing problems, the audience also got to see a good blend of theory and applications. The manuscript [3]—by Prof. Friggstad and his collaborators—documents the combinatorial algorithm mentioned above.

Davoud Abdi (University of Calgary) closed the scientific program with his presentation of exciting developments in poset theory related to the Cantor-Schröder-Bernstein Theorem. He introduced *NE-free* posets, classified them, and sketched a new proof of a 'Thomassé Conjecture' for countable NE-free posets; see [1].

3 Meeting outcomes

This workshop was originally scheduled for May 2020 (as BIRS 20w2245), then postponed to September 2021, and finally to 3–5 June 2022. (Phew! We finally met!)

The original conception was to bring together Alberta and Montana researchers in the fields of Combinatorics and Algorithms **in a student-friendly environment**. The organizers hoped this would foster collaboration, community, and networking opportunities, especially for junior researchers.

Anecdotally, the event was successful in these goals.

One participating graduate student, 'Terry Doe', related a telling story to a workshop organizer. A few weeks before this BIRS event, s/he had attended another conference, where s/he decided "I guess math conferences are not for me." Fast-forward those weeks to see a dramatic about-face. As BIRS was unfolding, Terry completely reevaluated her/his position and expressed the hope that this same workshop would run again in 2023. "I don't know if it's that I like being around Canadians or what, but I feel so welcome here!"

At meals and other breaks, the organizers heard from many of the other attendees how much they were enjoying the workshop. There was uniformly excellent quality in the scientific program. And the relaxed atmosphere of Banff was conducive both to casual conversation and to mathematical inquiry. The organizers know of several instances of near-future collaborative visits initiated exclusively because of this BIRS workshop.

Two constructive critiques were voiced: (i) schedule fewer talks per day to allow for informal problem sessions; (ii) try for a one–two days longer event to allow for more sustained collaborative interaction.

These suggestions were welcomed by the organizers, who overall heard nothing but praise for a well-planned and structured two-day workshop.

Selected references

[1] Davoud Abdi, Claude Laflamme, Atsushi Tateno, and Robert Woodrow, An example of Tateno disproving conjectures of Bonato-Tardif, Thomassé, and Tyomkyn, arXiv:2205.14679 [math.CO], 2022.

[2] Michael Cavers and Karen Seyffarth, Reconfiguring vertex colourings of 2-trees, *Ars Math. Contemp.* **17** (2019), 653–698.

[3] Sina Dezfuli, Zachary Friggstad, Ian Post, and Chaitanya Swamy, Combinatorial algorithms for rooted prize-collecting walks and applications to orienteering and minimum-latency problems, arXiv:2111.07414 [cs.DS], 2021.

[4] Ryan B. Hayward (with Bjarne Toft), *Hex: the Full Story*, CRC Press, Boca Raton, 2019.

[5] Ryan B. Hayward, *Hex: a Playful Introduction*, Vol. **54** in Anneli Lax New Mathematical Library, MAA Press, Providence, 2022.

[6] P. Mark Kayll and C.E. Larson, Egerváry graphs: Deming decompositions and independence structure, arXiv:2205.10598 [math.CO], 2022.

[7] Hadi Kharaghani, Sho Suda, and Vlad Zaitsev, On a class of optimal constant weight ternary codes, arXiv:2109.14995 [math.CO], 2019.

[8] Harmony Morris and Joy Morris, On generalised Petersen graphs of girth 7 that have cop number 4, arXiv:2009.00699 [math.CO], 2020.

[9] Cory Palmer and Dömötör Pálvölgyi, At most 3.55^n stable matchings, arXiv:2011.00915 [math.CO], 2020.