BIRS 20w5088: Interactions of gauge theory with contact and symplectic topology in dimensions 3 and 4 (online)

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1   Introduction

This workshop is the sixth in a series of BIRS "Interactions" workshops, with earlier meetings having taken place in 2007, 2009, 2011, 2013, and 2016. The talks covered recent results in gauge theory, contact and symplectic geometry, and low-dimensional topology. Each of these subjects is an active area of current research and interactions between them have led to breakthroughs on many long standing problems. The schedule included time for group work with break-out rooms after the talks. This encouraged lively discussion and allowed for a rich exchange of ideas.

As result of the pandemic, the meeting was conducted in an online format. While some were disappointed they were not able to travel and visit the Banff area, the new format had the benefit of allowing for the inclusion of many more mathematicians. Indeed, the sheer number of confirmed participants (155 people!) is evidence of the popularity of this meeting.

Apart from not meeting in person, one other notable difference was that we hosted fewer talks. This was a deliberate, and it was done in recognition of the challenges of parenting in the face of stay-at-home orders, as well as the difficulty of finding suitable times for such a large number of participants distributed over so many different time zones. As a result, the workshop featured an average of only two talks per day. In recognition of the #ShutDownAcademia #ShutDownSTEM #Strike4BlackLives movement, we did not schedule any talks on June 10.

In our survey of participants, we asked them "What are the main takeaways from the virtual delivery format? What worked, what didn't?"

According to the replies, the main advantage was that many more people could attend, and BIRS may choose to continue making the meetings open via Zoom even after BIRS resumes in person meetings. The main disadvantage was that it is more difficult to engage in a dialogue with the speaker or with other participants in the virtual format. As one of the respondents wrote, "The main disadvantage is that a virtual meeting lacks the 'electricity' (or 'energy' or 'vitality') of an in-person gathering

2   Brief overview of subject area

Over the last several decades it has become clear that the topology of manifolds in dimensions 3 and 4 is subtly and beautifully intertwined with various flavors of geometry (hyperbolic, symplectic and contact),

as well as with ideas from physics, such as gauge theories and topological quantum field theories. Collaborations among people working in these diverse areas have exploded over the last ten years, resulting in the remarkable solutions to long-standing conjectures in topology as well as the birth of entire new sub-fields and perspectives. Some of the more spectacular recent results include the solution to the triangulation conjecture for higher dimensional manifolds, the proof that Khovanov homology detects the unknot, characterizations of fibered knots in terms of Floer homologies, etc.

Critical to these developments has been the information provided by a vast array of new invariants whose definitions were motivated by gauge theory and topological quantum field theory. These invariants – Donaldson-Floer, Seiberg-Witten, Ozsváth-Szabó, Khovanov homology, Embedded Contact Homology to name a few – have intriguing relations among them, and a better understanding has led to significant progress on key problems in geometric topology, contact and symplectic geometry, and mathematical physics. An even more promising direction is the interplay between these invariants and more constructive approaches to low-dimensional manifolds – open book decompositions of contact 3-manifolds, symplectic fillings, Lefschetz fibrations, surgery constructions among many others. This interaction between powerful invariants and constructive methods is now more than ever a major driving force in the subject.

3 Highlights from the Workshop

The workshop featured ten talks on recent results in gauge theory, contact and symplectic topology, including:

- the Atiyah-Floer conjecture,

- unification of various Floer-theoretic invariants,

- spectral sequences relating Floer and Khovanov homologies,

- bordered and sutured versions of Floer-type invariants, and

- adjunction inequalities and applications.

In particular, the talks on the Atiyah-Floer conjecture (Daemi) and on the rectangular peg problem (Greene) were striking for the fresh ideas that were brought to bear and the many related open questions. Lambert-Cole's talk was also an impressive for his proof of the adjunction inequalities using topological/symplectic techniques instead of gauge theory.

4 Featured Talks

Below is a detailed list of speakers, titles, and brief descriptions of their talks.

1. **Matt Hedden** (Michigan State) *Relative adjunction inequalities and their applications*
   This talk discussed joint work [HR1, HR2] with Katherine Raoux on proving relative adjunction inequalities and applying them to define concordance invariants of links in a general setting, and to prove new results about contact structures, to motivate a 4-dimensional interpretation of tightness, and to show that knots with simple Floer homology in lens spaces (or L-spaces) minimize rational slice genus.

2. **Robert Lipshitz** (Oregon) *Khovanov homology detects split links*
   This talk discussed joint work [LS] with Sucharit Sarkar giving a proof, in terms of the Ozsváth-Szabó and Kronheimer-Mrowka spectral sequences, that the module structure on Khovanov homology detects split links.

3. **Vera Vertesi** (Vienna) *Bordered contact invariants*
   This talk discussed using foliated open books to extend the definition of the contact invariant for bordered Floer homology and applications to questions in low-dimensional topology.

4. **Peter Lambert-Cole** (Georgia Tech) *Symplectic trisections and the adjunction inequality*
   This talk discussed trisections for symplectic 4-manifolds with applications including a new proof of the adjunction inequality as well as new results on the minimal genus problem [LC1, LC2]

5. **Kristen Hendricks** (Rutgers) *Rank inequalities for the Heegaard Floer homology of branched covers*
   This talk discussed joint work [HLL] with T. Lidman and R. Lipshitz giving a proof that, for any nullhomologous knot K in a 3-manifold Y, there is a spectral sequence relating the Heegaard Floer homologies of Y with its 2-fold cover branched along K, along with a rank inequality for $\widehat{HF}$.

6. **Joshua Greene** (Boston College) *The rectangular peg problem*
   This talk discussed joint work [GL1, GL2] with Andrew Lobb giving a solution of the rectangular peg problem. This asserts that for every smooth Jordan curve and rectangle in the Euclidean plane, one can place four points on the curve at the vertices of a rectangle similar to the one given. The proof utilizes techniques of symplectic geometry in a surprising way.

7. **Aliakbar Daemi** (Washington University in St. Louis) *Lagrangians, $SO(3)$-instantons and the Atiyah-Floer Conjecture*
   This talk discussed joint work with Kenji Fukaya and Maksim Lipyanskyi on a variant of the Atiyah-Floer conjecture relating the framed Floer homology (as defined by Kronheimer and Mrowka) to symplectic framed Floer homology (as defined by Wehrheim and Woodward).

8. **Jonathan Hanselman** (Princeton) *Knot Floer homology as immersed curves*
   This talk discussed a new approach to understanding knot Floer homology via decorated immersed curves in the torus, along with applications to problems in Dehn surgery such as the cosmetic surgery conjecture.

9. **Juanita Pinzon-Caicedo** (Notre Dame) *Instanton and Heegaard Floer homologies of surgeries on torus knots*
   This talk discussed joint work with Tye Lidman and Christopher Scaduto relating the instanton and Heegaard Floer homologies of 3-manifolds obtained by Dehn surgery on torus knots.

10. **Artem Kotelskiy** (Indiana) *The earring correspondence on the pillowcase*
    This talk discussed joint work [CHKK] with G. Cazassus, C. Herald and P. Kirk on Lagrangian correspondences in pillowcase homology induced by the earring tangle.

5   Scientific Progress Made

The workshop created a virtual community with a nice mix of graduate students, early career researchers, and more senior members. The talks included many exciting new developments in the field. One purpose of the interactions workshops is to help develop new collaborative relationships as well as to support longer-term collaborative projects. Another goal was to encourage early career mathematicians.

One of the questions in our survey was "How well did it (the meeting) encourage early-career mathematicians?" One younger mathematician wrote in response that he "found the conference encouraging and the mathematics exciting." Another wrote "I benefitted from talking to other participants in the discussion sessions after the talks, where interesting questions and ideas were communicated." A mid-career participant replied that "the main value for me was getting to meet some of the young people in the field who I might otherwise have seen at a conference, and getting more feedback on my work from young and senior people." Another replied that "A good number of the talks were by early-career mathematicians, which was good."

These are tall orders for an online workshop. This is one of the distinct advantages of holding in-person workshop in Banff. We tried to find an alternative way to encourage collaborative research, and we decided to propose a conference proceedings for the first time in the series of "Interactions" workshops. These efforts are currently underway, but many participants have expressed an interest in contributing their work, and we have secured a commitment from Geometry and Topology Monographs to publish the volume. We are currently soliciting research and expository articles, which will be reviewed for inclusion and eventual publication in the forthcoming "Interactions" volume.

References

[AFHLPV] Akram Alishahi, Viktória Földvári, Kristen Hendricks, Joan Licata, Ina Petkova, and Vera Vértesi, *Bordered Floer homology and contact structures*, **arXiv:2011.08672**.

[CHKK] Guillem Cazassus, Christopher M. Herald, Paul Kirk, and Artem Kotelskiy, *The correspondence induced on the pillowcase by the earring tangle*, **arXiv:2010.04320**.

[DK] Aliakbar Daemi and Kenji Fukaya, *Atiyah-Floer conjecture: a formulation, a strategy to prove and generalizations*, **arXiv:1707.03924.**

[GL1] Joshua Greene and Andrew Lobb, *The rectangular peg problem*, **arXiv:2005.09193**.

[GL2] Joshua Greene and Andrew Lobb, *Cyclic quadrilaterals and smooth Jordan curves*, **arXiv:2011.05216**.

[H] Jonathan Hanselman, *Heegaard Floer homology and cosmetic surgeries in S^3*, **arXiv:1906.06773**.

[HRW] Jonathan Hanselman, Jake Rasmussen, and Liam Watson, *Heegaard Floer homology for manifolds with torus boundary: properties and examples*, **arXiv:1810.10355**.

[HW] Jonathan Hanselman and Liam Watson, *Cabling in terms of immersed curves*, **arXiv:1906.06773**.

[HR1] Matthew Hedden and Katherine Raoux, *Knot Floer homology and relative adjunction inequalities*, **arXiv:2009.05462**.

[HR2] Matthew Hedden and Katherine Raoux, *4-dimensional aspects of tight contact 3-manifolds*, **arXiv:2010.13162**.

[HLL] Kristen Hendricks, Tye Lidman, and Robert Lipshitz *Rank inequalities for the Heegaard Floer homology of branched covers*, **arXiv:2007.03023.**

[LC1] Peter Lambert-Cole, *Lecture notes on trisections and cohomology*, **arXiv:2009.11263.**

[LC2] Peter Lambert-Cole, *Symplectic trisections and the adjunction inequality*, **arXiv:2009.06028.**

[LV] Joan E. Licata and Vera Vertesi, *Foliated Open Books*, **arXiv:2002.01752.**

[LPS] Tye Lidman, Juanita Pinzon-Caicedo, and Christopher Scaduto, *Framed instanton homology of surgeries on L-space knots*, **arXiv:2003.03329**.

[LPZ] Tye Lidman, Juanita Pinzon-Caicedo, and Raphael Zentner, *Toroidal homology spheres and SU(2)-representations*, **arXiv:2101.02621**.

[LS] Robert Lipshitz and Sucharit Sarkar, *Khovanov homology also detects split links*, **arXiv:1910.04246.**