LOCALLY FINITE LIE ALGEBRAS

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Alexander Zalesskii (University of East Anglia, UK)

August 30, 2003–September 5, 2003

1 The Motivation and the Goals of the Workshop

Being infinite objects, locally finite Lie algebras are very complicated in comparison with Lie algebras of finite dimension. Not only because the direct limits of finite-dimensional simple Lie algebras are much harder than thier constituent algebras, but also since not every simple locally finite Lie algebras can be represented as a limit of semisimple finite dimensional algebras. Thus, in addition to “classical” methods of Lie Theory in order to successfully study locally finite Lie algebras, it is important to attract methods from other areas of mathematics. These ideas are essentially “in the air” because the other classes of locally finite algebras and the class of locally finite groups have been around for quite a long while.

The comparison with the theories of locally finite groups and associative algebras seems especially fruitful. The latter theory has close connections with C*-algebras, and we believe that the wealth of the methods of this latter theory can be applied to the study of locally finite Lie algebras. The classification result by Baranov - Zhilinski for diagonal embeddings of simple finite-dimensional Lie algebras provides just one example of a successful application of the associative methods to the theory of locally finite Lie algebras. But much more research is needed to get a better understanding of more general locally finite Lie algebras.

The theory of locally finite groups is also a very well developed subject with a number of excellent contributions by many distinguished mathematicians.The book by Kegel - Wehrfritz summarizes an early stage of development of this theory. A number of conferences have been held on this topic in the past, including a large conference in Turkey in 1996. The theory of locally finite groups is very rich with methods, but so far there have been very few attempts to apply them to locally finite Lie algebras. One of the most fruitful tools so far turned out to be the notion of an inductive system of representations introduced by Zalesskii.

Finally, it should be mentioned that in the last decade a number of important papers have been published on locally finite Lie algebras, devoted to various aspects, such as root systems, representation theory, structure theory, etc. It was very interesting to survey these results, estimate the progress made, formulate new conjectures and discuss the feasibility of their solution.

By holding a workshop on the topic, there seemed to be a very good chance of giving a boost to the subject. The goal of the meeting was to bring together people from all those different areas mentioned above, to compare the results and approaches, and to devise new strategies for studying locally finite Lie algebras.

It should be mentioned in conclusion that Canada is a country with excellent traditions in Lie theory, developed by a number of good researchers both in mathematics and physics. So it was only very appropriate to hold the conference suggested on the Canadian soil.

Finally we remark that a somewhat smaller workshop on locally finite Lie algebras was held as a part of the sequence of Californian Lie Theory Workshops early in 2001 at University of California - Los Angeles.

2 Preparation Period

The preparation work for the workshop "Locally Finite Lie Algebras" started in 2001 when the group of organizers was formed and the first tentative decisions had been made concerning the future participants. Then we applied with BIRS for a full 5-day workshop, with 40 participants. Later the same year we learned about an approval of our application, although in the form of a half 5-day workshop, with 20 participants.

During the preparation period the main effort was directed to forming the team of participants, which should be optimal as concerns the goals of the workshop. At times, we encountered problems arising from the financing principles of BIRS, according to which only the stay at the venue of the workshop had to be covered. At the same time, we had a number of participants from Europe for whom travelling such a long distance could be quite costly. Here we have to acknowledge some financial help from the Department of Mathematics and Statistics of the Memorial University of Newfoundland.

The second problem was to try to attract top people from the areas of mathematics, which are not immediately connected to the topic of the workshop but yet could be very important for introducing new methods and comparing the extent of the development of similar subjects but in different areas. This turned out to be quite successful and we had such a group of well-known people from outside Lie Theory, with genuine interest in locally finite Lie algebras.

The organisational problems of this preparation period had been successfully solved, thanks to the most efficient work of the PIMS personnel.

3 The List of Participants of the Workshop

1. Allison, Bruce (University of Alberta)
2. Bahturin, Yuri (Memorial Univ.)
3. Baranov, Alexander (University of Leicester)
4. Benkart, Georgia (Univ. Wisconsin - Madison)
5. Bryant, Roger (University of Manchester Institute of Science and Technology)
6. Dimitrov, Ivan (Queen's University)
7. Elliott, George (University of Toronto)
8. Glockner, Helge (Technische Universitat Darmstadt)
9. Grantcharov, Dimitar (University of Alberta)
10. Handelman, David (University of Ottawa)
11. Kegel, Otto (Freiburg University)
12. Leinen , Felix (Johannes Gutenberg-University Mainz)

13. Neeb, Karl-Hermann (Darmstadt University)
14. Neher, Erhard (University of Ottawa)
15. Penkov, Ivan (UC Riverside)
16. Pianzola, Arturo (University of Alberta)
17. Shumyatsky, Pavel (University of Brasilia)
18. Strade, Helmut (Univ. Hamburg)
19. Wolf, Joseph (University of California - Berkeley)
20. Zalesskii, Alexander (Univ. East Anglia)

4 Schedule of Lectures/Talks

August 31, 2003

1. Yuri Bahturin "Introduction to locally finite Lie algebras"
2. Georgia Benkart "Life on the Wedge (and Superwedge)"
3. Ivan Penkov "Recent advances in locally finite Lie algebras and superalgebras and some unsolved problems"
4. Helmut Strade "Locally finite Lie algebras over fields of arbitrary characteristic"
5. Ivan Dimitrov "Irreducible weight modules of $gl(\infty)$"

September 1, 2003

1. Alexander Baranov "Simple locally finite Lie algebras"
2. Otto Kegel "Highly transitive permutation groups"
3. David Handelmann "Classification of locally finite dimensional semisimple algebras and actions—survey and directions"
4. Joe Wolf "Double fibtration transforms"
5. Felix Leinen "Existentially closed locally finite groups and probable parallels in the area of locally finite Lie algebras"

September 2, 2003

1. Alexander Zalesskii "Classification of simple infinite dimensional Lie subalgebras of locally finite associative algebras"
2. George Elliott "Classification functors"

September 3, 2003

1. Erhard Neher "Locally finite root systems"
2. Bruce Allison "Covering Algebras of Lie Algebras"
3. Karl-Hermann Neeb "Approximating infinite-dimensional Lie groups by locally finite ones"

4. Helge Gloeckner "Direct limits of Lie groups, topological groups and topological spaces"
5. Pavel Shumyatski "Positive laws in fixed points"

September 4, 2003

1. Pianzola "Loop Algebras"
2. Bruce Allison "Lie Powers of Group Representations"
3. Dimitar Grantcharov "Direct limits of Lie groups, topological groups and topological spaces"

5 Abstracts of Lectures/Talks

1. **Bruce Allison, University of Alberta, Canada**

   Covering Algebras Of Lie Algebras

   In this talk, based on joint work with S. Berman and A. Pianzola, we will discuss loop algebras (also called covering algebras) that are constructed from a base algebra A and a finite order automorphism s of A. (When $s = 1$, the loop algebra is said to be untwisted.) In the case when the base algebra is a Lie algebra, loop algebras have been used to obtain realizations of affine Kac-Moody Lie algebras and, more generally, extended affine Lie algebras.

   In this talk, we describe some methods and results to decide when two loop algebras are isomorphic. These methods include an interpretation of loop algebras as forms of untwisted loop algebras. This interpretation allows us to apply tools from Galois cohomology in the study of the isomorphism problem.

2. **Yuri Bahturin, Memorial University of Newfoundland, Canada and Moscow State University, Russia**

   Introduction To Locally Finite Lie Algebras

   The aim of this talk is to give basic definitions and formulate some general results in the theory of locally finite Lie algebras. We will exhibit several characteristic examples and describe connections with locally finite groups and associative algebras.

3. **Alexander Baranov, University of Leicester, UK**

   Simple Locally Finite Lie Algebras

   Simple locally finite Lie algebras are subdivided into three classes: finitary, diagonal, and non-diagonal. Several characterizations of these classes will be given and various properties of the corresponding algebras will be discussed.

4. **Georgia Benkart, University of Wisconsin - Madison, USA**

   Life On The Wedge (And Superwedge)

   This talk will survey various constructions of Lie algebras and Lie superalgebras using exterior powers. Recent work on Lie (super)algebras graded by finite root systems and locally finite simple Lie (super)algebras will be featured. If time allows, some connections with the Tits construction of exceptional simple Lie superalgebras will be presented.

5. **Roger Bryant, University of Manchester Institute of Science and Technology, UK**

Free Lie Algebras As Modules For Groups

I shall describe some recent results concerned with the module structure of a free Lie algebra under the action of a group. Let G be a group, K a field and V a finite-dimensional KG-module. Let $L(V)$ be the free Lie algebra over K which has V as a subspace and every basis of V as a free generating set. The action of each element of G on V extends to a Lie algebra automorphism of $L(V)$. Thus $L(V)$ becomes a KG-module, and each homogeneous component $L^n(V)$ is a KG-submodule, called the nth Lie power of V. We consider the problem of determining the modules $L^n(V)$ up to isomorphism.

6. **Ivan Dimitrov, Queen's University, Canada**

Irreducible Weight Modules Of $gl(\infty)$

7. **George Elliott, University of Toronto, Canada**

Classification Functors

A classification functor is a functor, which distinguishes isomorphism classes. (Not too many seem to exist.) Abstract classification functors can sometimes be constructed by means of an intertwining argument—based on working with inner (or generalized inner) automorphisms. What is more difficult is to find a realization of an abstract classifying category in terms of a concrete category in which the morphisms are set maps—not just equivalence classes of such.

8. **Helge Glöckner, Technical University of Darmstadt, Germany**

Direct Limits Of Lie Groups, Topological Groups And Topological Spaces

An ascending sequence $G_1 \subseteq G_2 \subseteq \cdots$ of finite-dimensional Lie groups is called *strict* if each G_n is closed in G_{n+1}, and equipped with the induced topology. In fundamental investigations by L. Natarajan, E. Rodríguez-Carrington and J.A. Wolf (1991-94), certain conditions on such directed systems had been described which ensure that the direct limit exponential map is sufficiently well-behaved in order to serve as a chart around the identity for an infinite-dimensional Lie group structure on the direct limit group $G = \bigcup_n G_n$. In the first part of the talk, I'll describe an alternative approach which always allows G to be turned into a Lie group, without any extra conditions on the directed system. Instead of using the exponential map, tubular neighbourhoods are used to create compatible families of charts. As an application, in many cases countable-dimensional locally finite Lie algebras can be integrated to Lie groups.

In the second part of the talk, I'll report on work in progress concerning direct limits of infinite-dimensional Lie groups. I'll consider three typical classes of infinite-dimensional Lie groups which, algebraically, are direct limits of infinite-dimensional Lie groups: 1. countable weak direct products of Lie groups; 2. test function groups; 3. groups of compactly supported diffeomorphisms. Extending earlier work by N. Tatsuuma, H. Shimomura and T. Hirai (1998), I'll discuss the direct limit property of such Lie groups (which may be satisfied or not) in the categories of topological spaces, topological groups, smooth manifolds, and in the category of Lie groups.

9. **Dimitar Grantcharov, University of California - Riverside, USA, and University of Alberta, Canada**

On The Structure And Characters Of Weight Modules Of Lie Algebras And Superalgebras

In this talk we will present a method of studying weight modules of Lie superalgebras $\mathfrak{g}$ of type I, and $\mathfrak{g} = \mathbf{W}_n$. The method is based on Mathieu's result that every simple weight $\mathfrak{g}$-module M with finite weight multiplicities is obtained from a highest weight module $L(\lambda)$ by a composition Ψ of a twist and localization. We study the properties of the twisted localization Ψ and relate a Jordan-Hölder series of a highest weight module X with a Jordan-Hölder series of the module $\Psi(X)$. As a main application of the method we reduce the problems of finding a $\mathfrak{g}_0$-composition series and a character formula for all simple weight modules with central character χ to the same problems for simple highest weight modules with the same central character. Some of our results are new already in the case of a classical reductive Lie algebra $\mathfrak{g}$.

10. **David Handelman, University of Ottawa, Canada**

Classification Of Locally Semisimple Algebras

This is a survey talk. All algebras are associative. A *locally semisimple* algebra is a union of an increasing family of (finite dimensional) algebras over a field. If the field is algebraically closed, then Elliott's theorem asserts that the naturally ordered Grothendieck group with an additional datum is a complete invariant. The structure of the invariant itself is fairly well understood, completely if the algebra is simple; this is a consequence of Choquet theory and a result of Effros, Handelman & Shen.

When the underlying field is real closed, the classification of locally semisimple algebras is via a triple, corresponding to the two finite dimensional division algebras. If A denotes the algebra and $\mathrm{K}_0(A)$ is ordered Grothendieck group, then a complete invariant is given by the triple $\mathrm{K}_0(A) \to \mathrm{K}_0(A \otimes \mathbf{C}) \to \mathrm{K}_0(A \otimes \mathbf{H})$ as homomorphisms of ordered abelian groups, together with an additional datum. However, the structure for the invariant, that is, the classfication theory for the invariant itself is not completely understood (except when the algebra is simple).

If the underlying field is arbitrary (but perfect), a complete invariant for the algebras is known, but far from well understood. It extends the invariant of the real case, and involves all the finite dimensional division algebras that "appear" in the algebra.

Returning to the case that the underlying field be the complexes, there are analogous classification results for locally semisimple algebras with an action of a group, usually a locally representable action of a compact group. The invariants that result have close connections to random walks and limit ratio results.

11. **Otto Kegel, Freiburg University, Germany**

Highly Transitive Permutation Groups

12. **Felix Leinen, Mainz University, Germany**

Group Algebras of Simple Locally Finite Groups (*joint work with Orazio Puglisi*)

After an introduction to the various types of simple locally finite groups G we shall discuss the ideal lattice of their group algebra $\mathbb{F}G$ over a field $\mathbb{F}$ of characteristic zero. It was shown by

A. E. Zalesskiĭ, that the structure of such an ideal lattice is intimately related to the asymptotic behaviour of the $\mathbb{F}$-representations of the finite subgroups of G.

The ideal lattice turns out to be quite sparse in many cases. On the other hand, some tricky situations remain unsettled, especially when G is *approximated diagonally* by finite alternating groups.

Therefore we shall also discuss the convexly indecomposable normalized positive definite class functions $\mathbb{C}G \longrightarrow \mathbb{C}$ for certain direct limits G of finite alternating groups. These functions can be viewed as analogues of complex characters for G.

13. **Karl-Hermann Neeb, Technical University of Darmstadt, Germany**

    Approximating Infinite-Dimensional Lie Groups By Locally Finite Ones

    In the representation theory of infinite-dimensional Lie groups the following phenomenon occurs in many interesting situations: One is interested in an infinite-dimensional Lie group G containing a directed union of finite-dimensional subgroups which either is dense or at least "determines" in a certain sense the representations one is interested in. This establishes an interested link between certain direct limits of finite-dimensional groups and certain groups of operators on Hilbert spaces. The approximation by the finite-dimensional groups is crucial to determine the topology of the large group, its central extensions and, to some extent, also its representations

14. **Erhard Neher, University of Ottawa, Canada**

    Locally Finite Root Systems

    Locally finite root systems are defined in analogy to the definition of a finite root system, except that the finiteness condition is replaced by local finiteness, i.e., the intersection of the root system with every finite-dimensional subspace is finite. A theory of locally finite root systems has recently been developed by Ottmar Loos and the speaker (Weyl groups, bases, classification, parabolic subsets, positive systems, weights). In the talk, the basic structure theory will be presented. We will also consider Lie superalgebras graded by locally finite root systems, and give a characterization of the locally finite ones.

15. **Ivan Penkov, University of California - Riverside, USA**

    Recent Advances In Locally Finite Lie Algebras And Superalgebras And Some Unsolved Problems

    I will describe a class of semisimple locally finite Lie superalgebras admitting a root decomposition (classically semisimple locally finite Lie superalgebras) which contains the class of root reductive locally finite Lie algebras, and in particular $gl(\infty)$. Then I will describe all Cartan subalgebras of $gl(\infty)$. Finally, I will describe the current state of the theory of weight modules over root-reductive locally finite Lie algebras. Throughout the talk, I will state open problems.

16. **Arturo Pianzola, University of Alberta, Canada**

    Loop Algebras.

    The (twisted) loop algebras of finite dimensional simple complex Lie algebras were first considered by V. Kac to provide concrete realizations of the affine Kac-Moody algebras. I will begin by describing a procedure that allows one to view loop algebras in general in terms of (algebraic) principal homogeneous spaces. Several examples will be given to illustrate this point (which is very natural and geometric in nature).

The bulk of the talk will be centered around joint work with B. Allison and S. Berman on iterated loop algebras and related applications to the study of Extended Affine Lie Algebras.

If time permits, I will briefly talk about some applications of loop algebras to the classification of conformal algebras.

17. **Pavel Shumyatski, University of Brasilia, Brazil**

Positive Laws In Fixed Points

Let A be a finite group acting coprimely on a finite group G. It is well-known that the structure of the centralizer $C_G(A)$ (the fixed-point subgroup) of A has strong influence over the structure of G. The best illustration for this phenomenon is the fact that if G admits a fixed-point-free automorphism of prime order then G is nilpotent and the nilpotency class of G is bounded by a function depending only on the order of the automorphism (Higman-Thompson Theorem). Thus we see that in certain situations restrictions on centralizers of coprime automorphisms result in very specific identities that hold in G. An interesting problem is to describe as many such situations as possible. Powerful Lie-theoretic results of Zel'manov provide us with very effective tools for dealing with the problem. Those tools are employed to prove the following theorem.

Theorem. *Let q be a prime. Let A be an elementary abelian group of order q^3 acting on a finite q'-group G in such a manner that $C_G(a)$ satisfies a positive law of degree n for any $a \in A^{\#}$. Then the entire group G satisfies a positive law of degree bounded by a function of n and q only.*

The above theorem depends on the classification of finite simple groups which seems to be necessary to reduce the theorem to the case that G is nilpotent. Once this is done, Lie-theoric methods come into play and have crucial effect.

There are examples showing that the theorem fails if the group A has order q^2.

18. **Helmut Strade, Hamburg University, Germany**

Locally Finite Lie Algebras Over Fields Of Arbitrary Characteristic.

Many constructions of locally finite Lie algebras use direct limits of simple finite dimensional Lie algebras. In positive characteristic the Block-Wilson-Strade-Premet classification determines these simple Lie algebras in characteristic $p > 3$. The additional families (besides the classical ones) allow many more types of embeddings, and therefore give rise to new families of locally finite Lie algebras. All these algebras have not yet been investigated at all.

We shall give an overview on these and some other typical characteristic p phenomena.

19. **Joe Wolf, University of California - Berkeley, USA**

Double Fibration Transform

The double fibration transforms considered here, carry cohomology of holomorphic vector bundles to spaces of holomorphic functions, in a manner equivariant for the action of a semisimple Lie group. The best-known example is the complex Penrose transform. In the last year there has been a lot of progress on the general theory for the double fibration transform from holomorphic vector bundles on a flag domain. This talk is an indication of the current state of the theory.

20. **Alexander Zalesskii, University of East Anglia, UK**

    CLASSIFICATION OF SIMPLE INFINITE DIMENSIONAL LIE SUBALGEBRAS OF LOCALLY FINITE ASSOCIATIVE ALGEBRAS

    This is a joint work with A. A. Baranov and Yu. A. Bahturin. The main result is a kind of classification of simple Lie subalgebras in locally finite associative algebras over complex number field. In fact, we reduce the problem to the classification of simple associative algebras with involution as follows.

    Let L be a simple Lie subalgebras (under the bracket multiplication) of locally finite associative algebra A. Then either L is of finite dimension or there exists an associative algebra B with involution $*$ such that L is isomorphic to the commutator subalgebra $[U, U]$ of the Lie algebra U of $*$-skew symmetric elements of B (that is, $U = \{b \in B : b^* = -b\}$. Additionally, B has no non-zero proper $*$-stable ideal. In general B is not an envelope of L in A.

6 The Scientific Outcome of the Workshop

The main outcome of the workshop was the mutual recognition of the fact that the theory of locally finite Lie algebras is an interesting topic in mathematics with a whole number of important difficult problems concerning both the structure and the representation theories and having already a number of serious achievements. It became even more apparent that using methods of adjacent areas of locally finite associative algebras, locally finite groups and infinite-dimensional Lie groups is vital for the further progress in this area.

One of the surprising discoveries learned by the most of participants already during the workshop became the connection between simple Lie subalgebras of locally finite associative algebras and the root graded algebras. As noted by Bruce Allison, "It was interesting for me to learn of the connection between simple locally finite Lie algebras and root graded Lie algebras. I was surprised and interested to find out that there is a connection between the work of yourself [Bahturin], Zalesski and Baranov and the work of myself, Georgia and Yun Gao on BC graded algebras". Alexander Zalesskii writes: "I learned two things: connections between our work and Allison-Benkart-Gao, and connections between my work with I. Suprunenko on representations of algebraic groups in prime characteristic whose all weights are of dimension 1 and Benkart's work (with coauthors) on the similar problem for the highest weights infinite dimensional representations of Lie algebras in char 0 whose all weights are one dimensional. These links may lead to further interesting observations and research as I expect".

Another nice feature of the workshop was that in some talks a unification attempt was made for several theories mentioned above. This was especially noticeable in the talk of Otto Kegel who stood at the origin of many directions in Algebra. George Elliott, one of the key scholars in Operator Algebras, gave a talk about general classification principles, which have been or could be used in every theory, including locally finite associative and Lie algebras.

The results reported by the participants can be grouped as follows:

1. **Results on the classification of locally finite Lie algebras (and superalgebras).**

   The workshop clearly showed that the classification theory of locally finite simple associative and Lie algebras has reached a very mature state, and that many deep results on representations of locally finite simple algebras are available. The relevant talks are Baranov, Bahturin, Benkart, Penkov, Strade, Neher, Zalesskii. The classification results are the most exhaustive in the case of finitary Lie algebras of operators in infinite-dimensional space (Baranov, with Strade in the case of positive characteristic) and direct limits of simple finite-dimensional Lie algebras (Baranov - Zhilinskii). In the case of simple diagonal Lie algebras, which is the same as Lie subalgebras of locally finite associative algebras, the classification by Bahturin - Baranov - Zalesskii is actually a reduction to the classification of involution simple locally finite associative algebras. Some of the classification - like results in the the case of algebras over positive characteristic fields have been reported by H. Strade.

The root graded algebra approach to locally finite Lie algebras was elaborated in the talk of Georgia Benkart.

The results on the classification of locally finite root systems of Lie algebras and Lie superalgebras were described by Erhard Neher.

The main results of Penkov's was that any countably dimensional semisimple locally finite Lie superalgebra which admits a generalized root decomposition and is generated by the generalized root spaces is isomorphic to a direct sum of classical or exceptional simple Lie superalgebras and copies of $\mathfrak{sl}(m,\infty)$, $\mathfrak{sl}(\infty,\infty)$, $\mathfrak{osp}(m,\infty)$,$\mathfrak{osp}(\infty,\infty)$, $\mathfrak{osp}(\infty,2k)$, $\mathfrak{sp}(\infty)$ and $\mathfrak{sq}(\infty)$.

As already mentioned, a general approach to the classification problems was suggested in the lecture of George Elliott.

2. **Results on representations of locally finite Lie algebras and superalgebras as well as infinite dimensional Lie groups.**

   The relevant talks were roughly those by Penkov, Dimitrov, Benkart.

   The main result of Ivan Penkov's talk was a description of some invariants of weight modules over $\mathfrak{gl}(\infty)$. In particulat I showed an integrable irreducible module with finite shadow and no highest weight.

   Ivan Dimitrov gave a classification of all irreducille weight modules of $\mathfrak{gl}(\infty)$ with finte dimensional weight spaces.

   Georgia Benkart has constructed some irreducible modules over diagonal Lie algebras using their presentation as root graded algebras.

   It was pointed out that for many important classes of locally finite Lie algebras only the most basic representation theory is available, including module of the highest weight. But these algebras very often have only roots but no root vectors, or even no root systems at all. The question was asked what can be the sustitute of these notions in constructing the representation theory of such Lie algebras. The simplest example is a Lie algebra $L = \mathfrak{gl}(2^\infty)$ obtained as the direct limit of Lie algebras $L_n = \mathfrak{gl}(2^n)$, with the structure embeddings $X \to \mathrm{diag}\{X, X\}$, $X \in L_n$.

3. **Results on relations with associative algebras**

   Relevant talks are due to Baranov, Handelman, Zalesskii.

   Alexander Baranov presented some of his results on the diagonal locally finite Lie algebras, in particular his characterization of algebras, which satisfy "Ado's Theorem" about the possibility of embedding a locally finite Lie algebra into a locally finite associative algebra. It looks now that apart from locally finite Lie algebras, which have root decomposition, this is the best explored class with powerful methods and still a number of intriguing problems to solve. The diagonal Lie algebras have been touched upon also in the talks of Yuri Bahturin, Georgia Benkart, and Alexander Zalesskii.

   David Handelman gave a very illuminating talk, by invitation of the organizing committee, on the history and contemporary state of the classification theory of direct limits of semisimple finite-dimensional algebras, with or without additional structure, such as the action by groups. This talk is very important because, as mentioned, some classification problems of locally finite Lie algebras tirned out to be equivalent to those of associative algebras with involution.

4. **Results on the limits of Lie groups**

   Relevant talks are due to Neeb and Gloeckner.

   As noted by Karl - Hermann Neeb, almost nothing is known, in a systematic fashion, on representations of infinite-dimensional Lie groups. There are many interesting Lie groups G whose

Lie algebra $\mathfrak{g}$ contains a dense locally finite (simple) Lie algebra $\mathfrak{f}$, so that (unitary) representations of G automatically provide representations of $\mathfrak{f}$, in general by unbounded operators. It is an important problem to establish a link between the algebraic theory and the analytic (= Lie group) theory by determining systematically which representations of the Lie locally finite Lie algebra $\mathfrak{f}$ do arise from a representations of the global group G. Of course this depends heavily on the group G, so that one has to consider a subclass of representations of $\mathfrak{f}$ defined by the group G which should behave better than the "uncontrolled" representations of $\mathfrak{f}$.

This method has been applied in the work of G.I. Olshanski in the context of topological groups G and in the context of representations of Lie groups by J. Wolf and his coauthors and by K.-H. Neeb to direct limits of highest weight representations of complex simple Lie groups. These special instances show that there often is a rich structure behind the interplay between the global group G and the Lie algebra $\mathfrak{f}$. Typical examples where these techniques have not yet explored and seem very promising are "classical" Lie subgroups of unit groups of AF C^*-algebras.

Another interesting and very systematic talk on direct limits of finite or infinite-dimensional Lie groups was given by Helge Gloeckner, who showed a very important fact that in many cases countably dimensional locally finite-dimensional Lie algebras can be integrated into Lie groups. This of course immediately raises the questions of using this interaction to the benefit of both Lie groups and locally finite Lie algebras.

5. **Relations with non-locally finite Lie algebras**

   Relevant talks are due to Bruce Allison and Arturo Pianzola.

   These talks on loop algebras gave the participants a flavour of approaches and methods used in a neighbouring area of Infinite-Dimensional Lie algebras. Both authors mentioned that the intersection of these two classes is not very large but no doubt the cohomological methods suggested in their joint research are of great importance for locally finite Lie algebras.

6. **Relations with Group Theory**

   Relevant talks are due to Kegel, Leinen and Shumyatski.

   The lecture of Otto Kegel gave the participants a number of interesting examples of locally finite objects in groups, rings and Lie algebras and showed their connection to each other both mathematically and historically.

   The talk of Felix Leinen contained a lot of information about various classes of locally finite groups and their group rings. In the whole number of cases the lattice of ideals of such group algebras is extremely poor. Finding wide classes of locally finite Lie algebras whose universal algebras have the same property (no proper nonzero two sided ideals except the augmentation ideal) would be very interesting and the similarity of approaches via Zalesskii's inductive limit of representations gives hope that this can actually be done. The progress achieved in the theory of locally finite groups is very stimulating because this area famous by many difficult open problems.

   Pavel Shumyatski pointed out that using powerful results from Infinite-Dimensional Lie Theory has produced a number of interesting theorems in abstract group theory.

   In the talk of Roger Bryant the author duscussed the Lie powers of representations of finite groups.

7. **Talks about phenomena in the finite dimensional case which can be useful in the direct limit case**

   These interesting talks have been delivered by Joe Wolf and Dimitar Grantcharov

7 General Comments

- " . . . I enjoyed the conference very much. It gave a stimulating picture of a lively and developing area. . . "
- ". . . Overall, I was very pleased to learn about the field of locally finite Lie algebras. I was aware that there was activity in the field, but as a nonexpert this conference gave me an otherwise unavailable opportunity to find out the state of the area. . . "
- " . . . Many thanks for a very nice, enjoyable and stimulating meeting! There is much I learned and much I'll have to look up.. . . "
- " . . . Thank you so much for the wonderful conference. I got a lot out of it.. . . "