Developments in Superstring Theory

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At present, there are a number of important facets of string theory in which physics is related to deeper mathematics. These include the relationship between Yang-Mills theory and gravity (the AdS/CFT correspondence), work on matrix models and supersymmetric gauge theories, mirror symmetry and classification of string compactifications through techniques such as the derived category, topological string theory, and string compactifications with fluxes, with their relationship to cosmology. The workshop provided an opportunity to survey some developments on these subjects, and to catalyze further work in these directions. The following is a summary of talks given at the workshop.

Hirosi Ooguri gave a talk about the large N duality between topological open string theory and closed string theory. First he reviewed the conjecture by Gopakumar and Vafa about the equivalence between the Chern-Simons gauge theory and the closed string theory near the conifold singularity, and then he presented a field theoretical proof of this conjecture based on his work with Vafa. He discussed how this is related to the correspondence between the matrix models and the N=1 gauge theories in four dimensions. This involves explaining how to use topological string to compute F-terms in low energy effective theories of string compactifications, which was pointed out by Bershadsky, Cecotti, Ooguri, and Vafa. He also discussed meaning of non-planar diagrams of the matrix models as computing effects of the certain deformation of the corresponding gauge theories.

Michael Douglas of Rutgers University gave a review of recent work which shows that the exact superpotential of certian supersymmetric gauge field theories is computed exactly by a certain matrix integral. This is an interesting recent development which has many interesting implications in both mathematical and physical issues surrounding supersymmetric theories and has inspired a large amount of work on matrix models and their relationship with hierarchies of differential equations and integrable models.

Douglas also reviewed recent work with N. Seiberg and E. Witten on anomalies and the chiral ring structure in a supersymmetric U(N) gauge theory. They considered theories with an adjoint chiral superfield and an arbitrary superpotential. He argued that a certain generalization of the Konishi anomaly leads to an equation which is identical to the loop equation of a bosonic matrix model. He argued that this allows one to solve for the expectation values of the chiral operators as functions of a finite number of "integration constants." He showed that this fact can be used to derive the Dijkgraaf-Vafa relation of the effective superpotential to a matrix model. Some of the results are applicable to more general theories. For example, they can determine the classical relations and quantum deformations of the chiral ring of $\mathcal{N} = 1$ super Yang-Mills theory with SU(N) gauge group, showing, as one consequence, that all supersymmetric vacua have a nonzero chiral condensate.

Washington Taylor talked about some recent work on string field theory, in two directions:

a) Performing loop computations in bosonic string field theory, particularly the one-loop open string tadpole calculation, which was work recently published with Ellwood and Shelton. This result

shows that the open string tadpole can see the long range gravitational effects of a D-brane usually associated with the closed string tadpole, but confirms the expected picture that the quantum bosonic theory is problematic due to the closed string tachyon; his talk at Banff led to several useful discussions which helped improve the final understanding of the results in the published paper, which appeared a month or so after the meeting.

b) Computing the abelian and nonabelian vector field theories on a D-brane by explicitly integrating out the massive fields in open string field theory. This computation gives a systematic approach to deriving the abelian and nonabelian Born-Infeld actions as well as derivative corrections, and exemplifies the complications of a background-independent theory, which necessitates a complicated field redefinition to get the variables natural for a particular background. Again, several useful discussions at Banff improved their understanding of these results, which were recently published with Coletti and Sigalov.

Paul Aspinwall spoke on mirror symmetry, whihc forms a relationship between a pair of Calabi-Yau manifolds which appear to be unrelated classically. Originally this relationship was forged by considering closed strings propagating on these target spaces. More recently it has been realized that open strings, and consequently D-branes, give more detailed information of the nature of the mirror relationship. In particular, it demonstrates an association to the derived category conjectured by Kontsevich.

His talk focused on a notion of D-branes stability originally proposed by Douglas, Fiol and Romelsberger. This leads naturally to the idea of a triangulated category which manifests itself in a very different way for the two mirror Calabi-Yau manifolds. Many details of this picture are currently poorly understood.

Ezra Getzler discussed a deformation of the Toda chain, which is called the equivariant Toda lattice, described by a pair of Lax equations, which by the calculations of Okounkov and Pandharipande describes the integrable system associated to the equivariant Gromov-Witten theory of CP^1. This implies (and clarifies) the Toda conjecture of Eguchi, Hori and Yang, which is the non-equivariant limit.

Sheldon Katz's talk on "Geometric Transitions and D-branes" discussed the N=1 field theories arising from wrapping D-branes on the vanishing holomorphic cycles in an extremal transition, joint work with Cachazo and Vafa. A superpotential is generated classically. The vacuum moduli space was identified with the geometric moduli space of appropriate coherent sheaves. Other more recent developments were covered as well.

These results have been combined with large N duality to describe the dynamics of gaugino bilinears, which have since also been described via matrix model techniques.

Anton Kapustin gave a talk on his work on topological A-branes which live on coisotropic submanifolds. He provided string theory arguments which show that A-branes are not necessarily Lagrangian submanifolds in the Calabi-Yau: more general coisotropic branes are also allowed, if the line bundle on the brane is not flat. He showed that a coisotropic A-brane has a natural structure of a foliated manifold with a transverse holomorphic structure. And he argued that the Fukaya category must be enlarged with such objects for the Homological Mirror Symmetry conjecture to be true.

Tuesday's focus was on connections between compactifications and cosmology. There has been a growing consensus that recent cosmological data points to the prevalence of a positive vacuum energy, or cosmological constant. If string theory describes nature, we must find ways to understand this cosmological constant, and physics in spaces with a cosmological constant, within the framework of the theory.

Shamit Kachru reported on recent constructions of string theory vacua with positive cosmological constant. Although aspects of these compactifications are under investigation, they appear to represent first examples demonstrating existence of de Sitter vacua in string theory. The models are constructed from the starting point of orientifold, or more generally F-theory, compactifications with explicit three branes and fluxes present. The fluxes dynamically fix the complex structure moduli of the compactification. In cases where there is a single Kahler modulus, this is generally then fixed by non-perturbative corrections. This typically results in an anti-de Sitter vacuum, but he summarized recent work with Kallosh, Linde, and Trivedi pointing out that adding an anti-D3

brane could lift the vacuum energy to a positive value.

The resulting de Sitter vacuum is, however, only metastable, and will ultimately decay in a decompactification transition. **Steve Giddings** summarized recent work on a very general result of this nature. In particular if 1) there is a positive vacuum energy, as we've apparently now observed in nature and 2) there are extra compact dimensions, then the general result is that the present vacuum state of the Universe is unstable, typically to some form of decompactification transition. However, the lifetime for such a transition is expected to be extremely long as compared to the present age of the Universe.

Eva Silverstein reported on recent work focussed in part on the question of understanding de Sitter entropy in the context of D3 branes. In particular, she discussed the entropy of concrete de Sitter flux compactifications and deformations of them containing D-brane domain walls. She summarized the relevant causal and thermodynamic properties of these "D-Sitter" deformations of de Sitter spacetimes, and discussed a string scale correspondence point at which the entropy localized on the D-branes (and measured by probes sent from an observer in the middle of the bubble) scales the same with large flux quantum numbers as the entropy of the original de Sitter space, and at which Bousso's bound is saturated by the D-brane degrees of freedom (up to order one coefficients) for an infinite range of times. From the geometry of a static patch of D-Sitter space and from basic relations in flux compactifications, her work with Fabinger finds support for the possibility of a low energy open string description of the static patch of de Sitter space.

Wednesday included a talk by **Sergei Gukov**, which was based on recent work where he studied three-dimensional Chern-Simons gauge theory with complex gauge group. His main focus was SL(2,C) Chern-Simons theory, which has many interesting connections with three-dimensional quantum gravity and geometry of hyperbolic three-manifolds. In the talk he explained that, in the presence of a single knotted Wilson loop in an infinite-dimensional representation of the gauge group, the classical and quantum properties of such theory are described by an algebraic curve called the A-polynomial of a knot. In particular, quantization of the theory can be formulated in terms of the (Euclidean) effective quantum mechanics on a non-commutative torus, so that the partition function of SL(2,C) Chern-Simons theory in the stationary phase approximation is given by the semi-classical wave function supported on the zero locus of the A-polynomial.

Using this approach, one can obtain some new and rather surprising relations between the A-polynomial, the colored Jones polynomial, and other invariants of hyperbolic 3-manifolds. These relations generalize the volume conjecture and the Melvin-Morton-Rozansky conjecture, and suggest an intriguing connection between the SL(2,C) partition function and the colored Jones polynomial.

Eric Sharpe also discussed modelling D-branes with sheaves, and in particular, a powerful computational method that arises when thinking about D-branes in such a mathematical framework. Specifically, he showed how to see directly in BCFT that open string spectra between D-branes corresponding to sheaves are counted by Ext groups. This result has been assumed by various authors for a number of years, and has even been checked in massive theories believed to flow in the IR to the BCFT's in question, but has never previously been understood directly in BCFT. Verifying in BCFT that open string spectra are calculated by Ext groups turns out to involve interesting physical realizations of spectral sequences. He also briefly discussed some permutations of this problem, namely D-branes in flat B fields (where one has a twisted gauge bundle, instead of an honest gauge bundle), and also D-branes in orbifolds, and showed how one sees Ext groups in each of these contexts also.

Thursday began with **Andreas Karch**, who spoke on large N dualities, which have played an important role both in mathematics and physics. In the physical context so far the only regime that is understood is the one of strong coupling on the field theory side (the open string side) which is related to the supergravity limit of the closed string theory. To go beyond this, a fist step is to investigate the limit where the field theory is well understood: the free field theory. Actually one can use the open string side as the definition of the theory and see to what extend it can be rewritten as a closed string theory. He demonstrated that the a free scalar field in light cone frame can be rewritten to become a closed string theory on AdS in the zero radius limit. The surprise is that the closed string worldsheet has to be discretized.

Finally, **Shiraz Minwalla** of Harvard University discussed the some features of the thermody-

namic phase diagram of $\mathcal{N} = 4$ supersymmetric Yang-Mills theory and its string theory dual, type IIB superstring theory.

The workshop provided a first-class atmosphere for interaction between speakers and for research, and was well received.

Work performed at the conference included Douglas finishing the paper "The statistics of string/M theory vacua," JHEP 0305 (2003) 046, which discusses a new approach to the study of superstring compactification. He had many useful conversations on this with participants at the workshop.

There were a number of other positive comments. For example, H. Ooguri commented "It was a great conference. I learned a lot by talking to participants, both physicists and mathematicians. The facility at Banff is very nice and the location is wonderful. I hope there will be more conferences like this. Thank you for organizing it." W. Taylor commented "On the whole, I found the Banff workshop to be one of the best physics environments I had attended in some time. Everything was well-organized, the facilities were very good, and the number and quality of people at the workshop was, in my opinion, close to optimal. It was a great workshop, and I hope it happens again!"

In closing the organizers would like to thank the staff at Banff for the excellent facilities and support.