Representation theory of lineraly compact Lie superalgebras and the Standard Model

Alberto De Sole (Harvard University),
Victor Kac (M.I.T.),
Aleksey Rudakov (University of Tronheim),
Minoru Wakimoto (Kyushu University)

July 26-August 16, 2003

A linearly compact Lie algebra is a topological Lie algebra whose underlying space is a topological space isomorphic to the space of formal power series over $\mathcal{C}$ in finite number of variables with formal topology. Examples include the Lie algebra of formal vector fields W_n on an n-dimensional manifold M and its closed infinite-dimensional subalgebras. Cartan's list of simple linearly compact Lie algebras consists of four series: W_n and its subalgebras of divergence 0 vector fields, Hamiltonian vector fields and contact vector fields.

In the "super" case, i.e., when M is a supermanifold, the answer is much more interesting: there are ten series and also five exceptional Lie superalgebra of vector fields, denoted by $E(1,6)$, $E(4,4)$, $E(3,6)$, $E(3,8)$ and $E(5,10)$ [1].

Here comes a possible connection to the Standard Model: it turns out that the maximal compact subalgebras of $E(3,6)$ and $E(5,10)$ are $K = su_3 \times su_2 \times u_1$ and su_5, respectively, whereas the corresponding compact Lie groups are the groups of symmetries of the Standard and the Grand Unified Model respectively. Of course, K uniquely embeds in su_5, and it turned out that this embedding extends to the embedding of $E(3,6$ in $E(5,10)$. Moreover, the "negative part" of $E(5,10)$ as a su_5 module decomposes with respect to K precisely into the multiplets of leptons and quarks as described by the Standard Model.

In [2] representation theory of $E(3,6)$ was developed, and some further observations were made on its connections to the Standard Model. In [3] some initial progress was made on representation theory of $E(5,10)$.

The program consisted of mathematics and physics parts:

I. Mathematics part.

   First we reviewd the known results on representation theory of $E(3,6)$ and $E(5,10)$ and connections between them. Next, we found new singular vectors for $E(5,10)$ as compared to [3] and made some progress in proving that there are no other singular vectors. We are hopeful that the methods we developed in BIRS will lead to a complete representation theory of $E(5,10)$. We also hope that a complete representation theory of $E(3,6)$ and $E(5,10)$ and connections between them will shed a new light both on the Standard Model and the Grand Unified Model.

II. Physics part.

We had a general review on quantum field theories and the Standard Model [4]. The topics covered in the review sessions are:

1. Lagrangian and propagator in free field theories: free boson, free fermion and free vector field.
2. Gauge invariance in QED: local $U(1)$-invariance, Ward identities, Faddeev–Popov ansatz.
3. Non abelian gauge theories: Yang–Mills Lagrangian, Faddeev–Popov ansatz and ghost fields.
4. Spontaneous symmetry breakdown: Higgs mechanism.
5. Grand unified theories.
6. Possible interpretation of the exceptional infinite dimensional Lie superalgebras $E(3,6)$ and $E(5,10)$ as "hidden" symmetries of a quantum field theory.

References

[1] V. Kac, Classification of infinite-dimensional simple linearly compact Lie superalgebras, Adv. Math. 139(1998), 219-272.

[2] V. Kac and A. Rudakov, Representations of the exceptional Lie superalgebra $E(3,6)$ II Four series of degenerate modules.

[3] V. Kac and A. Rudakov, Complexes of modules over exceptional Lie superalgebras $E(3,8)$ and $E(5,10)$, IMRN 19(2002),1007-1025.

[4] S. Weinberg, Quantum field theory.