Topological Bifurcations in the Wake Behind an Oscillating Cylinder

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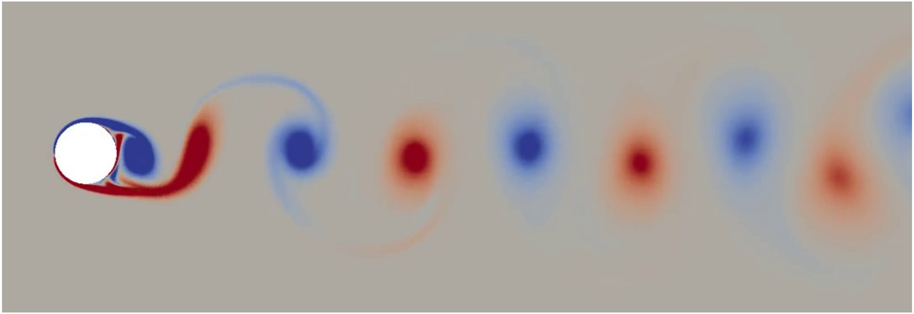
INDEPENDENT RESEARCH FUND DENMARK

von Kármán vortex street

2S wake

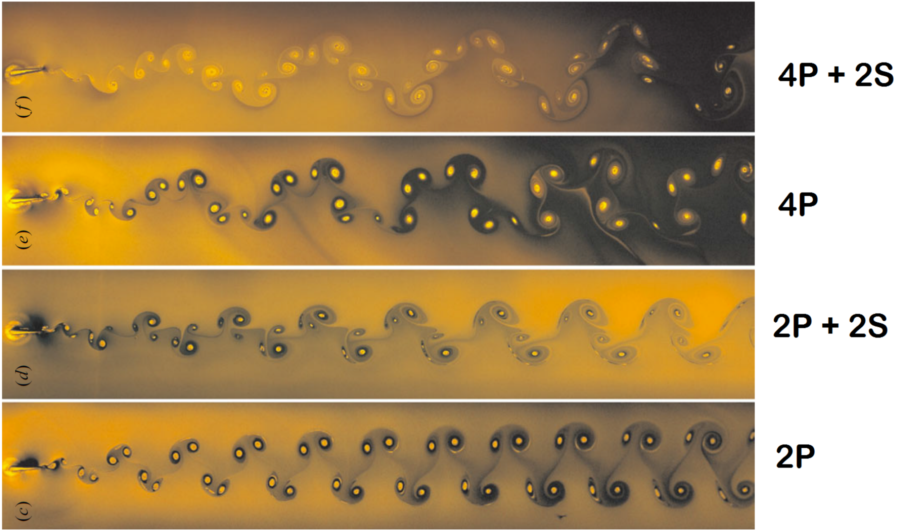


Oscillating cylinder may lead to exotic wakes

P + S wake

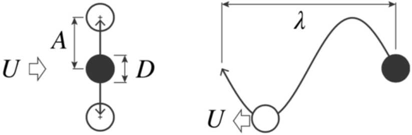
Williamson, in Ponta & Aref, J. Fluids Struct. 22(2006), 327–344

Other body shapes produce very exotic wakes



Schnipper *et al*, J. Fluid Mech. 633(2009), 411–423

Forced transverse oscillations of the cylinder



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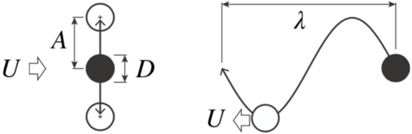

Dimensionless parameters

- $Re = \frac{UD}{\nu}$ (= 100 here)
- $A \leftarrow \frac{A}{D}$
- $f \leftarrow \frac{f}{f_{st}} = \frac{T_{st}}{T}$ or
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where f_{st} (T_{st}) is the frequency (period) of the vortex shedding for $A = 0$

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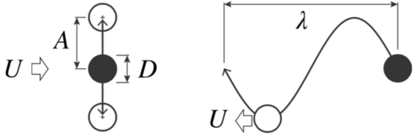

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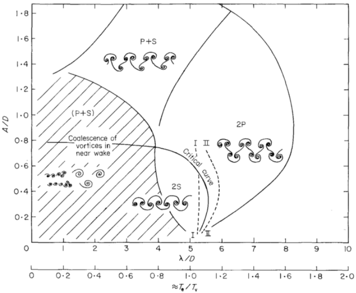
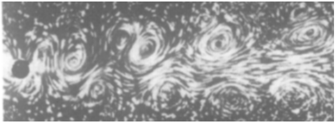
Williamson & Roshko,
J. Fluids Struct. 2(1988), 355-381





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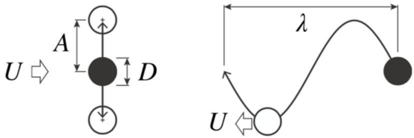
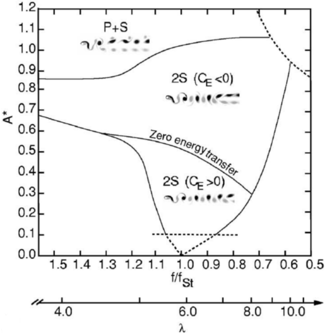


Leontini, Stewart, Thompson & Hourigan, Phys. Fluids 18(2006), 067101



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Purpose of the present study

To elucidate the transitions in the structure of the wake from a 2S to a P+S pattern as the forcing amplitude A is varied, at $f = f_{st}$ and $Re = 100$.

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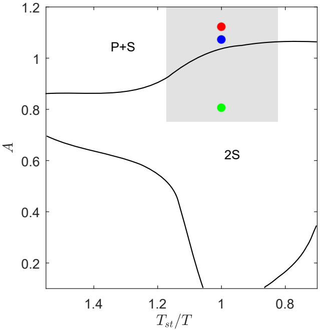
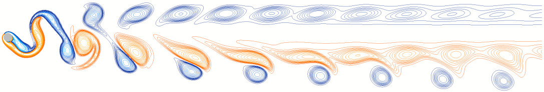
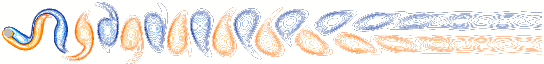


Leontini *et al.* (2006)

- $A = 1.120$

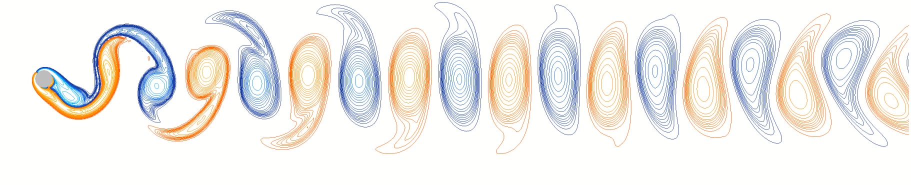


- $A = 0.800$



Locating the vortices, their creation, and disappearance

- $A = 1.092$: note the symmetry $\omega(x,y,t) = -\omega(x,-y,t+1/2)$



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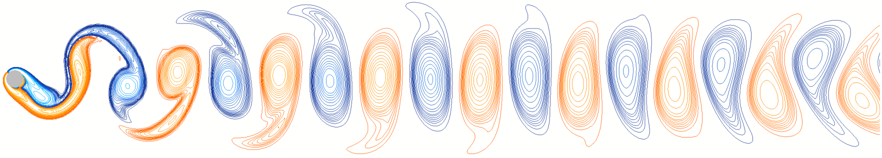

Vortices:

$$\partial_x \omega(x^*,y^*,t^*;A) = 0$$
$$\partial_y \omega(x^*,y^*,t^*;A) = 0$$
$$|\mathbf{H}^\omega(x^*,y^*,t^*;A)| > 0$$

Saddle points:

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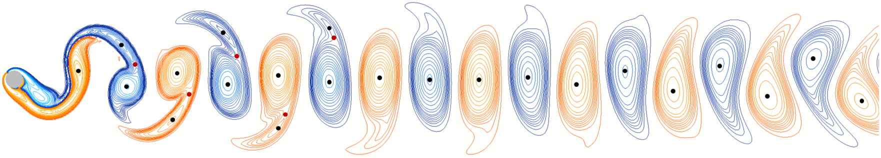

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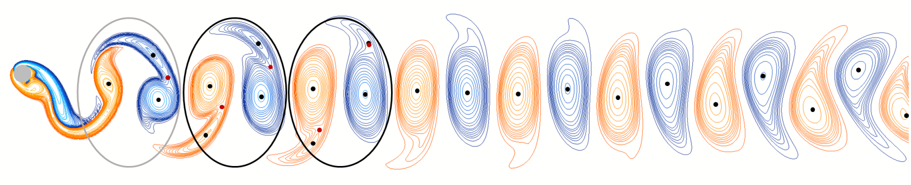
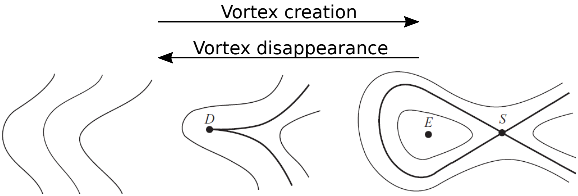


An extremum and a saddle are created or disappear in a *cusp bifurcation* at a degenerate critical point of vorticity where the Hessian is singular.



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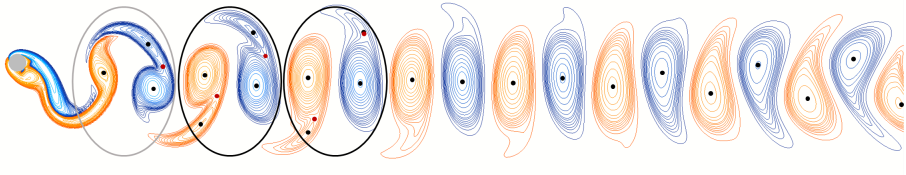

- Extended classification concatenated by symbols Σ^n with $\Sigma = 2\text{S}, \text{P}+\text{S}, 2\text{P}, \ldots$.
- Integer n designates the number of periods $+1$ the pattern exists.

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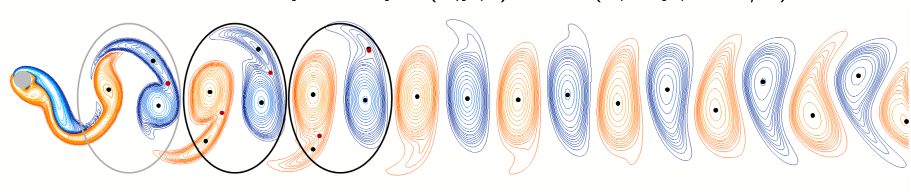

This wake is classified as $(2P)^3(2S)^\infty$

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Bistability

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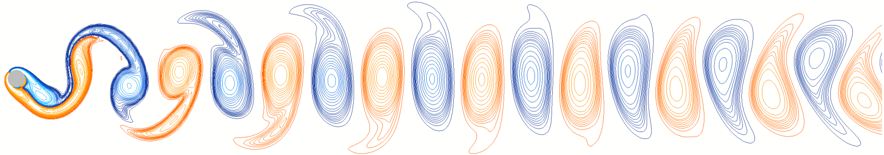

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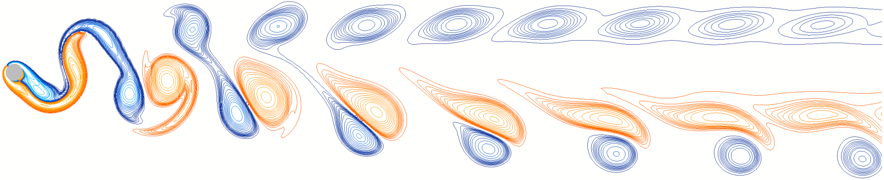

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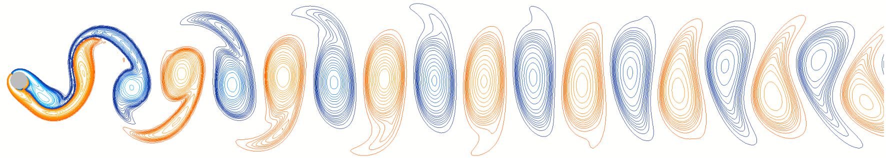
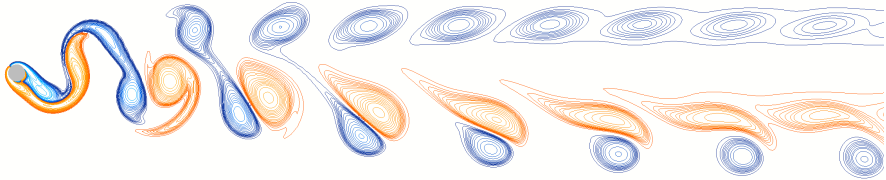


- $\omega(x,y,t) \neq -\omega(x,-y,t+1/2)$

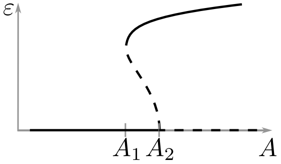


A dynamical subcritical bifurcation

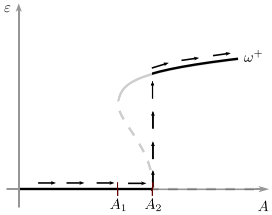
Matharu, Hazel & Heil, J. Fluid Mech. 918(2021), A42:

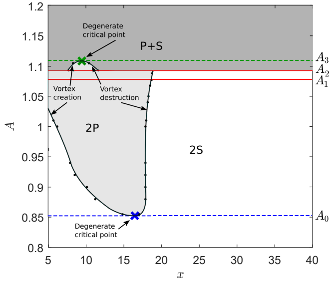
- A subcritical symmetry-breaking pitchfork bifurcation from the symmetric branch occurs at $A_2 = 1.093$
- The bifurcating branch gains stability at a fold bifurcation at $A_1 = 1.078$

$$\varepsilon = \|\omega(x,y,t) + \omega(x,-y,t+1/2)\|$$



Topological bifurcation diagrams





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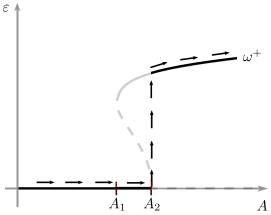
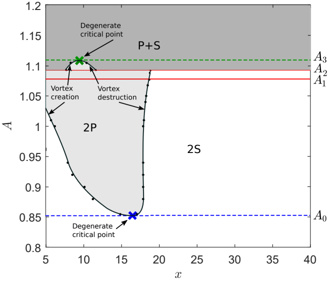
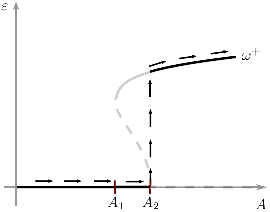
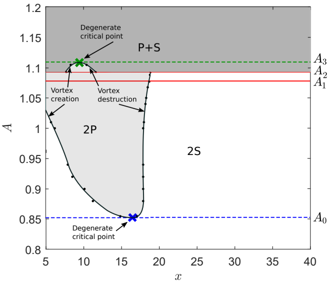




$$A < A_0 : (2S)^\infty = 2S$$

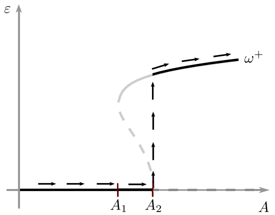
Topological bifurcation diagrams

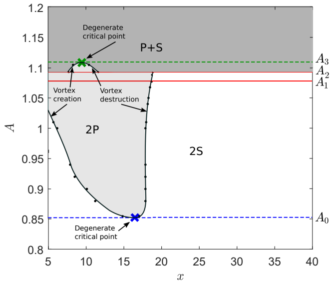




$A_0 < A < A_2 : (2S)^n(2P)^m(2S)^\infty$

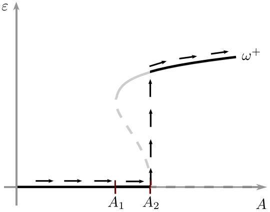
Topological bifurcation diagrams

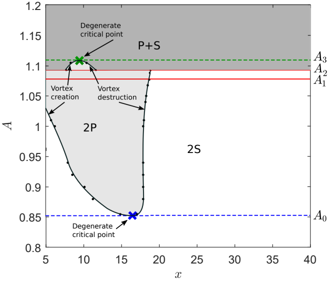




$$A_2 < A < A_3 : (\mathrm{P}+\mathrm{S})^n(2\mathrm{P})^m(\mathrm{P}+\mathrm{S})^\infty$$

Topological bifurcation diagrams





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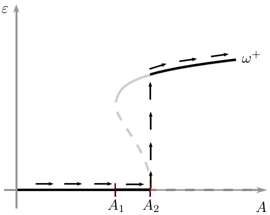

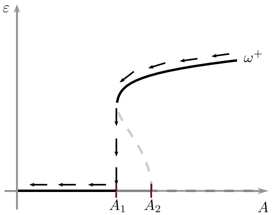

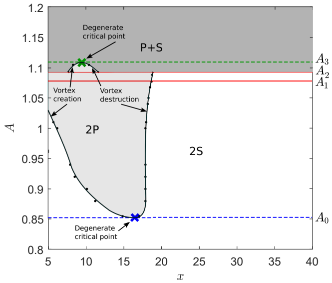

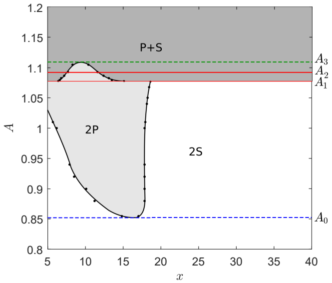

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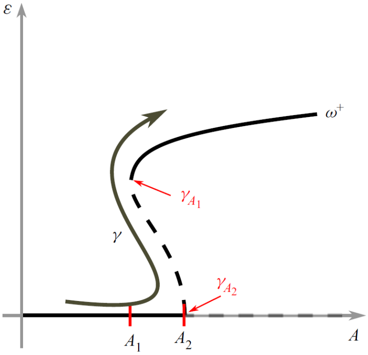
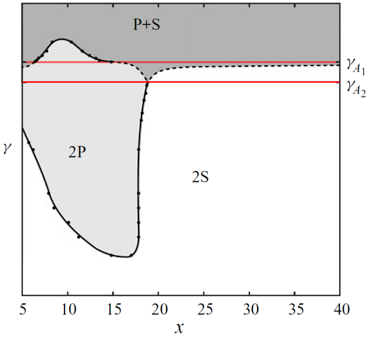
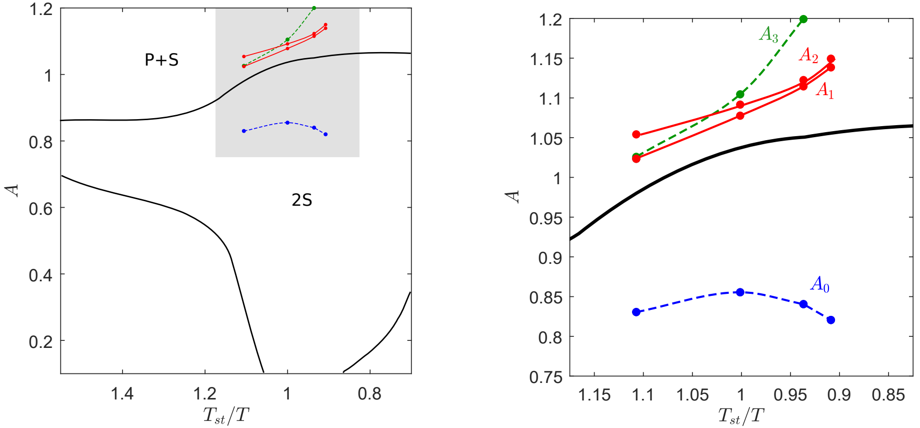
Following the unstable branch





Varying the forcing period



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Details in AR Nielsen, PS Matharu & M Brøns: *Topological bifurcations in the transition from two single vortices to a pair and a single vortex in the periodic wake behind an oscillating cylinder*, J. Fluid Mech. 940(2022), A22.