

“Alberta Number Theory Days XIV” 23w2006 (Banff)

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Website: <https://www.birs.ca/events/2023/2-day-workshops/23w2006>

1 Overview of the Field

Number theory is a broad and central area of research with many connections and applications to other areas of mathematics and science. Canada is one of the major hubs for number theory in the world, and western Canada has always been one of the major Canadian centres for number theory. Alberta number theorists are working in a wide range of sub-disciplines, including analytic number theory, arithmetic geometry, automorphic forms, and the Langlands programme, computational number theory, algebraic dynamics and diophantine geometry, and the interactions of number theory with mathematical physics and with applications in cryptography.

2 Scientific Highlights

Alberta number theorists are working in a wide range of sub-disciplines, including analytic number theory, arithmetic geometry, automorphic forms, and the Langlands programme, computational number theory, arithmetic statistics, algebraic dynamics and diophantine geometry, and the interactions of number theory with mathematical physics and cryptography.

There were a total of 13 presentations given: 4 one hour plenary lectures and 9 twenty minute lectures. 12 of these lectures were delivered in person from the Juniper Hotel in Banff and 1 was delivered online. Out of all presentations, 11 video recordings were shared to the public. Our aim was to actively include a number of early-career researchers, post-doctoral fellows, and graduate students, in our speaker list. We also asked speakers for their talks to be graduate-student-friendly.

2.1 Plenary Lectures

We invited 4 speakers from outside of Alberta to give presentations on their research. These speakers were carefully selected: it was important to ensure that the research presented fell under the umbrella of research topics supported by Alberta institutions. The 4 speakers (and approximate research area) were Alina Bucur (arithmetic statistics), Avi Kulkarni (arithmetic geometry), Matilde Lalin (analytic number theory), and Bella Tobin (arithmetic statistics). The titles and abstracts of their talks are included here:

- **Alina Bucur (University of California San Diego) **Virtual Talk****

Title: *Counting D_4 quartic extensions of a number field ordered by discriminant*

Abstract: A guiding question in number theory, specifically in arithmetic statistics, is counting number fields of fixed degree and Galois group as their discriminants grow to infinity. We will discuss the history of this question and take a closer look at the story in the case of quartic fields. In joint work with Florea, Serrano Lopez, and Varma, we extend and make explicit the counts of extensions of an arbitrary number field that was done over the rationals by Cohen, Diaz y Diaz, and Olivier.

- **Avi Kulkarni (Dartmouth College) Juniper Talk**

Title: *Nonarchimedean integral geometry*

Abstract: In this talk, I will discuss a nonarchimedean integral geometry formula for the action of a compact K -analytic group on a homogeneous space. This formula is analogous to a result over the reals obtained by Howard. Some applications will be discussed.

(Joint work with Antonio Lerario and Peter Burgisser.)

- **Matilde Lalin (University of Montréal) Juniper Talk**

Title: *The distribution of values of cubic L -functions at $s = 1$.*

Abstract: We investigate the distribution of values of cubic Dirichlet L -functions at $s = 1$. Following ideas of Granville and Soundararajan, and Dahl and Lamzouri for quadratic L -functions, we model values of $L(1, \chi)$ with the distribution of random Euler products $L(1, \mathbb{X})$ for certain family of random variables $\mathbb{X}(p)$ attached to each prime. We obtain a description of the proportion of $|L(1, \chi)|$ that are larger or that are smaller than a given bound, and yield more light into the Littlewood bounds. Unlike the quadratic case, there is a clear asymmetry between lower and upper bounds for the cubic case.

(This is joint work with Darbar, David, and Lumley.)

- **Bella Tobin (Oregon State University) Juniper Talk**

Title: *Applications of Belyi Maps in Arithmetic Dynamics*

Abstract: Unicritical polynomials, typically written in the form $z^d + c$, have been widely studied in arithmetic and complex dynamics and are characterized by their one finite critical point. The behavior of a map's critical points under iteration often determines the dynamics of the entire map. Rational maps where the critical points have a finite forward orbit under iteration are called postcritically finite (PCF), and these are of great interest in arithmetic dynamics. The family of (single-cycle normalized) dynamical Belyi polynomials have two fixed critical points, so they are PCF by construction. These maps provide a new testing ground for conjectures and ideas related to post-critically finite polynomials and using this family, we can begin to explore properties of polynomials with two critical points. In this talk we will discuss how the family of dynamical Belyi polynomials connects to the more general setting of bicritical polynomials and how we can use it to classify PCF polynomials, to answer a question of Silverman pertaining to the height of critically fixed maps, and to determine reduction properties of PCF maps. In particular, we will demonstrate that considering maps with only one additional critical point may be enough to provide complete answers to questions in arithmetic dynamics. Much of the work discussed is joint with Michelle Manes and Jacqueline Anderson.

2.2 Junior research highlights

We invited junior speakers from each institution who had not had the opportunity to share their research in a previous ANTD meeting. These talks exhibited the breadth of research being conducted in Alberta: topics included Analytic, Algebraic, Computational Number Theory, Automorphic Forms, and Representation Theory. We scheduled 9 shorter talks (20 minutes plus 5 for questions) given by 6 postdocs, 2 advanced PhD students talking about their current work, and 1 MSc student finished in summer 2023.

- **Ertan Elma (U. Lethbridge PDF), Juniper talk**

Title: *A Discrete Mean Value of the Riemann Zeta Function and its Derivatives*

- **Punya Plaban Satpathy (U. Alberta PDF), Juniper talk**

Title: *A Borel-Serre type compactification for loop groups*

- **Félix Baril Boudreau (U. Lethbridge PDF), Juniper talk**

Title: *Arithmetic Rank Bounds for Abelian Varieties*

- **Mishty Ray (U. Calgary PhD student), Juniper talk**

Title: *Geometry of local Arthur packets and Vogan's conjecture for GL_n*

- **Andrew Schopieray (U. Alberta PDF), Juniper talk**
Title: *Numerical invariants of fusion*
- **David Marquis (U. Calgary PhD student), Juniper talk**
Title: *Computing Class Groups of Quartic Number Fields Using Sieving*
- **Yanze Chen (U. Alberta PDF), Juniper talk**
Title: *Theta liftings for loop $GL(n)$ groups*
- **Gulizar Sedanur Albayrak (U.Calgary PDF), Juniper talk**
Title: *Quantitative estimates for the size of an intersection of sparse automatic sets*
- **Solaleh Bolvardizadeh (U.Lethbridge Masters student), Juniper talk**
Title: *On the quality of the ABC-solutions*

The question sessions were very active and many continued into the breaks/scheduled social times. It seems that many collaborations had formed during this weekend conference.

3 Comments

3.1 Facilitating communication among participants

The scheduled BIRS coffee breaks and designated social time in the evenings allowed participants to meet, talk about current positions and goals, and discuss research. The organizers took turns chairing the sessions, splitting morning and afternoon sessions the first day and the morning of the second. This included the facilitation of questions from both in person attendees and online attendees. There was a small but very consistent and active online presence and those who had questions were able to ask the speakers directly.

3.2 Fostering a welcoming environment at BIRS

The BIRS staff included a land acknowledgement in their opening remarks. This is something that the organizers asked BIRS to deliver at the last ANTD meeting in 2021 and we are happy that this has continued and was not something that we had to ask for explicitly. This workshop was hosted at the Juniper hotel which provided a different experience for those who had been to BIRS in the past. There were some issues with meeting space and while these were not largely impactful to the grand goals of the conference we wanted to list them here. The organizers had to repeatedly ask the staff at the Juniper for access to the meeting room on the day of our arrival. As the Juniper is relatively small there was no good place for participants to have a meet and greet, we figured the meeting room could be used for this as was discussed with BIRS before the conference started. It did, however, take multiple attempts and phone calls to the general manager to eventually gain access to this room. Additionally, as we had 6 observers (self funded participants) who joined for group meals: the main issue is that the meal prices for these observers were higher than the menu price which led to some confusion.

Besides these small comments the conference was an overall success. The BIRS staff were extremely helpful and were able to get us answers to almost all questions we had over the course of the weekend.

4 Outcome of the Meeting

Like previous years, the XIVth edition of Alberta Number Theory Days was a welcome opportunity to display the quality of talent nurtured in Calgary, Edmonton, and Lethbridge and to connect these groups with researchers outside of Alberta. Most speakers were students and postdocs who presented their work, often for the first time outside their institution, and they got feedback from an interested and welcoming audience.

This year we had four plenary lectures given by both early career researchers and professors. These speakers were from institutions outside of Alberta but whose research area is supported by the Alberta institutions: the goal of bringing in researchers from outside of the province was to increase the research network for Alberta but also to expose graduate students and early career researchers to people working in their field.

We hoped these connections would result in additional collaborations and research opportunities for number theorists in Alberta.

Out of all presentations, 11 video recordings were shared with the public, including three of the plenary lectures.

ANTD is traditionally a vital part to keep Alberta's expanding research groups in number theory cohesive while geographically apart. This year the event returned to an almost entirely in-person format. We did, however, invite a number of virtual participants and ended up having a small but consistent and engaged online audience. This allowed for participation of graduate students at the Alberta institutions, and also drew in some participants from other PIMS institutions and beyond. After a couple of years of completely online/hybrid events we recognize the importance of events like ANTD in building a welcoming and supporting community in number theory in Western Canada. We made the decision to skip one year (2022) in hopes that the format would return to mostly in-person and, while we were excited to offer online opportunities to those that we could not invite, we were happy to have such a strong and diverse showing of number theory being done in Alberta and to have this meeting almost entirely in-person again.