Mexican Mathematicians in the World: Perspectives and Recent Contributions (18w5142)

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1 Overview of the Meeting

The conference "Mexican Mathematicians in the World" connected Mexican mathematicians residing abroad with their counterparts in Mexico by showcasing the current research of promising and well-established Mexican mathematicians, and by fostering focused, in-depth discussions about different aspects of the overall situation of Mathematics in Mexico.

This conference aimed to raise the academic profile and visibility of Mexican mathematics within the international scientific community, and to strengthen the individual and institutional links among Mexican mathematicians and Mexican institutions.

2 Outcomes and Achievements of the Meeting

The main objectives of the workshop were addressed as follows.

1. **Expand efforts to promote interaction among Mexican mathematicians around the world.** The meeting format, consisting of 18 scientific talks, 3 poster sessions, 3 organized panels/ discussion forums and an outreach talk for the general public, all surrounded by plenty of interaction time during meals and coffee breaks, allowed for numerous interactions and networking among workshop participants, who came from more than 10 countries and 10 Mexican states.

2. **Disseminate the scientific work of Mexican mathematicians abroad.**
   There were scientific talks presented by Mexican mathematicians pursuing scientific careers in various countries in the Americas and Europe. Presentation highlights are detailed in Section 3.

   During the workshop, scientific talks were recorded using BIRS video facilities. These talks will remain available for participants and other interested scientists at the workshop website
   `https://www.birs.ca/events/2018/5-day-workshops/18w5142`.

   The speakers were encouraged to contribute a paper to the internationally refereed proceedings of the meeting. In view of the positive response, we expect to publish the corresponding volume in the near future.

3. **Foster collaboration and academic exchange among Mexican mathematicians working abroad and their peers in Mexico.**
   In addition to having a group of participants speaking about their research, we organized three poster sessions, open to all participants (based in Mexico and abroad) and allowed for generous time for discussions. These sessions were very well attended and gave rise to extended research discussions. Apart from the written comments from participants presented in Section 4, various junior and senior participants verbally pointed out to organizers how such activities contributed to achieving the academic exchange and collaboration objectives.

4. **Exchange experiences concerning studying and working abroad.**
   A moderated discussion was organized around this subject. The discussion was guided by questions from interested students. Workshop participants living abroad generously shared their experiences, as well as tips and suggestions for those students planning to apply for and/or pursue graduate studies abroad.

5. **Analize opportunities for returning to Mexico.**
   A round-table was organized around this topic (Movilidad, Intercambio, Reincorporación), featuring recognized leaders of the Mexican mathematical and scientific community: Alejandro Adem (University of British Columbia University), Daniel Juan (Centro de Ciencias Matemáticas, UNAM), Víctor Rivero (Centro de Investigación en Matemticas), Gelasio Salazar (Universidad Autónoma de SLP and former President of the Mexican Mathematical Society) and Julia Tagüeña (CONACYT).

   An open forum took place with the participation of directors of a couple of departments of mathematics Humberto Gutiérrez (Universidad de Guadalajara) and Silvia Reyes (Universidad Tecnológica de la Mixteca), as well as early career researchers recently reincorporated to Mexican institutions: Ana Rechman (Instituto de Matemáticas, UNAM, CDMX), Felipe García Ramos (CONACYT - Universidad Autónoma de SLP), Alfredo Nájera (CONACYT - UNAM, Oaxaca) and Gerardo Hernández (UNAM, Juriquilla).

6. **Build and strengthen networks of global cooperation.**
   In addition to the above, a new team of organizers was formed, incorporating experi-

enced and newly recruited members, to follow up with the organization of a similar event in 2020. A database of Mexican mathematicians living abroad is being maintained by this group, with the contribution of participants and other members of the Mexican mathematics community.

3 Presentation Highlights

The workshop included 18 research talks (in Spanish) covering a range of current research activity of the participants. Here is a summary of the highlights.

- Alejandro Uribe, University of Michigan, Ann-Arbor, USA spoke about microlocal and semiclassical analysis. His talk included an introduction to microlocal and semiclassical analysis. These theories study the connections between functions and linear operators, on one side, and objects in symplectic geometry on the other side. The talk addressed recent results and applications of these theories.

- Carlos Hugo Jiménez Gómez (PUC-Rio, Brasil) presented some connections between functional inequalities and volume inequalities in convex geometry. This talk reviewed how various fundamental inequalities in analysis, including Sobolev, Log-Sobolev and Gagliardo-Niremberg inequalities, can be obtained using classical inequalities from convex geometry. The latter are usually concerned with parameters associated to convex sets in R^n, such as volume, surface area or diameter. The connection is not always obvious and this talk reviews some examples of how it occurs. The talk was based on recent works with J. Haddad y M. Montenegro [8, 9, 4].

- Victoria Cantoral Farfán (International Centre for Theoretical Physics, Italia) talked about the conjecture of Mumford-Tate for Abelian varieties. This talk presented the Mumford-Tate conjecture, which provides an analogy between algebraic groups defined over Q or Q_p for prime numbers p. This conjecture establishes a bridge between the celebrated Hodge conjecture (for complex varieties), and its arithmetic analogue, the Tate conjecture (for Abelian varieties defined over a number field). The talk described the three conjectures through various examples, and presented new results in the direction of Mumford-Tate conjecture.

- Enrique Treviño (Lake Forest College, USA) talked about the minimum quadratic residue and related problems. Let n be a positive integer. A number $q \in \{1, 2, ..., n-1\}$ is called a quadratic residue if there exists $x \in \mathbb{N}$ such that $x^2 = q(mod p)$; otherwise q is a non-quadratic residue. This talk addressed the problem of finding the order of magnitude of the minimum non-quadratic residu, and related problems such as the minimal inert prime in a real quadratic field.

- Araceli Bonifant (University of Rhode Island, USA) presented a discussion of simple examples of the Deligne-Mumford compactification.

- Jorge Castillejos (KU Leuven, Bélgica) talked about A non-commutative topological dimension. C* algebras are a type of sub-algebras of the algebra of bounded operators over a Hilbert space. In the commutative case, a C* algebra is isomorphic to

the space of continuous functions over a locally compact Hausdroff space. For this reason, C* algebras are considered non-commutative topological spaces. This talk presented a notion of non-commutative topological dimension, which revolutionized the area and provided a fundamental ingredient in the recent classification of C* algebras.

- Eduardo Dueñez (Spelman College, USA) discussed the topic of metastable convergence of ergodic average from the continuous logic viewpoint. This talk revisited certain classical and recent results on convergence of averages of a fixed element f of a topological vector space V endowed with an action $(g, f) \mapsto gf$ of an amenable (semi)group G. (In the special case when G = N is the semigroup of naturals, the averages are just $({}^1f +^2 f + \cdots +^n f)/n$). Such results, collectively called ergodic convergence theorems–although there is really nothing "ergodic" about them–, include the classical ergodic theorem of Birkhoff as well as von Neumann's mean ergodic theorem (MET), alongside subsequent generalizations. In collaboration with J. Iovino [6, 5], we use continuous logic to obtain a radically elementary proof of a MET valid for any polynomial action of an amenable group on a Hilbert space. The Compactness Theorem from logic implies the existence of universal rates of metastable convergence that depend only on the degree of the action.

- Jaime Santos Rodríguez (Universidad Autónoma de Madrid, España) talked about syntetic curvature and isometries. In the 1980s, Gromov defined a distance between Riemannian manifolds, modulo isometries. He demonstrated that the class of Riemannian manifolds with Ricci curvature bounded below is precompact. In the 1990s, Cheeger and Colding studied properties of the limits of sequences of such Rimannian manifolds. In 2006, Lott-Sturm-Villani defined a notion of syntetic Ricci curvature for spaces which are not necessarily manifolds. This condition is based on optimal transport between probability measures, and the convexity of an entropy functional. Spaces satisfying this condition are called CD(K,N) spaces and include Riemannian manifolds with Ricci curvature bounded below. This talk described examples and properties of these spaces, and the structure of their isometry groups [7].

- Ixchel Dzohara Gutiérrez Rodríguez (Universidad de Santiago de Compostela, España) presented on Einstein metrics on 4-dimensional manifolds. An important problem in Riemannian geometry is that of constructing a distinguished metric on a manifold. In the case of surfaces, one can consider metrics of constant Gauss curvature. An immediate question is: how to construct distinguished metrics for higher dimensional manifolds? This talk discussed Einstein metrics and some of their generalizations, based on [1, 2].

- Alicia Prieto Langarica (Youngstown University, USA) talked about modelling the effect of temperature on sleep quality. Sleep is a universal biological process which is still not well understood. One of the major questions is the relation between temperature and sleep quality. Experimental results suggest that environmental temperature changes may affect sleep patterns. This talk discussed a mathematical model, describing sleep dynamics and various characteristics of the REM/NREM cycle, which

can be helpful to better understand the relation between environmental temperature and sleep quality.

- Paola Vera Licona (UConn Health, USA) discussed the problem of set intersections and its applications. Let $S = \{S_1, \ldots, S_n\}$ be a family of sets. The intersection set T of S is a set which intersects each of the sets $S_i, I = 1, \ldots, n$. T is a minimal intersection set (MIS) if T does not contain a proper subset which is itself a MIS for S. The problem of generating the collection of MISs for a given family is of interest diverse research areas and has been studied (under diverse names) in combinatorics, Boolean algebra and computational biology. While some interesting results have been obtained for the associated decision problem, the computational complexity of the MIS problem remains unknown. This talk presented different algorithms for enumerating MISs and its computational performance in problems derived from various areas of scientific research, with an emphasis on computational systems biology.

- Daniel Ballesteros Chávez (Durham University, UK) talked about the non-linear problem of prescribed k-curvature in hyperbolic surfaces. The prescribed curvature problem in hypersurfaces has a close connection with some totally non-linear elliptic differential equations. In the case of Gaussian curvature, the so-called Minkowski problem led to the development of solutions of Monge-Ampere equations. This talk presented the problem of k-symmetric curvature in two types of ambient space, Riemannian and Lorentz, and discuss a priori estimates which are useful to establish existence and regularity of solutions.

- Alejandra Herrera (University of British Columbia, Canada) talked about identifying unique observations in superresolved microscopic images via a spatio-temporal model. STORM is a super-resolution technique that uses photoswitchable fluorophores to achieve resolutions at or below 20nm. A downside of STORM is the possibility of recording several blinks from one fluorophore, affecting the estimation of the number of molecules detected in the image. This talk presented a mathematical model to identify unique fluorophores in STORM images by independently using the localization and the time series of the observations. The temporal sequence is described with a Markov chain approach and their spatial distribution with a Gaussian mixture model. The speaker stated: "To estimate the parameter values, I implemented a maximum likelihood procedure which requires a mixed optimization. Initially, I solved the mixed optimization problem with an extensive search algorithm on integers and a continuous optimizer for the rest of the parameters. I am currently investigating MCMC and Bayesian methods to speed up the optimization. I have tested my protocol in simulated data and I will use it to improve STORM images of B-cell surface receptors. B-cell receptors distribution on the membrane has been related to B-cell activation. This model will enhance a microscopy technique that is already widely used in biological applications and will allow to deeper analyze immune cells signaling."

- Tonatiuh Sánchez-Vizuet (Courant Institute of Mathematical Sciences, USA) talked about the hybridizable discontinuous Galerkin method and an application to mag-

netic equilibrium in fusion reactors. In axisymmetric fusion reactors, the equilibrium magnetic configuration can be expressed in terms of the solution to a semi-linear elliptic equation known as the Grad-Shafranov equation, the solution of which determines the poloidal component of the magnetic field. Solving this problem efficiently, quickly and with high precision turns out to be important for the design of reactors, and for real time monitoring of plasmas in experimental settings. The Hybridizable Discontinuous Galerkin is a numerical solution strategy which, based in a weak formulation of the equation, turns the problem in a set of local subproblems. These subproblems can be solved in parallel, and the global solution is obtained by "glueing" the local solutions. This strategy can have a high order of approximation and it is very robust with respect to geometric properties of the domain. After introducing the basic ideas o the method, the talk presented an application to the Grad-Shafranov equation.

- Hildeberto Jardón, (Technical University Munich, Germany) talked about singularly perturbed ordinary differential equations (SPODEs). These equations have been studied for a long time, and have a range of applications including modelling of systems and natural phenomena with various timescales. Mathematical theory has been developed for the study of SPODEs, including asymptotic expansions, non-standard analysis and geometric theory of singular perturbations. This talk addressed applications and limitations of the latter, especiallly in the context of the appearance of singularities. There was also a discussion of a technique to resolve or de-singularize the dynamics in the neighborhood of a singularity, as well as other current research topics and perspectives in the context of singular perturbations.

- Javier Chávez Domínguez (University of Oklahoma, USA) talked about isoperimetric and Sobolev inequalities on magnetic graphs. The plane isoperimetric problem, which dates from a long time ago, aims at finding the region with the largest area, keeping the perimeter fixed. It is a classic theorem in analysis that the solution to this problem is closely related with the so-called Sobolev inequalities, which compare the norm of a function with the norm of its derivative in certain spaces of p-integrable functions. Sometimes the domains of interest are not continuous regions but discrete sets. A specially useful model is one in which the domain is a graph: a set of vertices and edges connecting some of them. Isoperimetric inequalities in this context are particularly relevant in Computer Science, as they play a crucial role in algorithm design and, surprisingly, they are also related to Sobolev inequalities. In some cases, for instance when there is a magnetic potential in certain quantic atomic models, in order to describe the system, it is necessary to know not only the graph, but also a complex number of modulus one associated to each edge. This talk presented isoperimetric inequalities for these "magnetic" graphs, which in turn yield Sobolev type inequalities.

- Diego Corro Tapia (Karlsruhe Institute of Technology, Germany) talked about Positive Ricci curvature and high-dimensional torus actions. In differential geometry, the study of Riemannian manifolds with positive sectional curvature is a classic topic. However, there are few known invariants which restrict the existence of such metrics.

The mean sectional curvature is called Ricci curvature. At first sight, the problem of endowing a manifold with a Riemannian metric of positive Ricci curvature should be simpler or more manageable than that of finding a metric with positive sectional curvature. However, this has not been the case. This talk discussed this topic, presenting results of [3], and giving examples, both implicit and explicit, of manifolds admitting a metric of positive Ricci curvature and a large prescribed symmetry group.

- Alfredo Hubard (Université Marne-La Vallée, France) gave a talk entitled "Dressing-up surfaces". He addressed the problem of graphs and metrics on surfaces, with the systolic inequality as a starting point. This inequality ensures that in every surface there is a not-too-long non-contractible curve. The speaker explained constructions, analogous to pairs of pants decompositions, which maximize the length of such curve. The following question was discussed: Given a surface S, is there a metric m such that every graph G which may be topologically embedded in S can be embedded in such a way that each edge is the shortest path between its endpoints?

4 Comments from Participants regarding Scientific Progress and Networks

- Tonatiuh Sánchez Vizuet (Courant Institute of Mathematical Sciences, New York University, USA) comments: "*The workshop provided an excellent place for establishing connections with peers and senior colleagues.* "

- Manuel Alejandro García Acosta (Facultad de Ciencias, Universidad Nacional Autónoma de México) comments: "*The workshop "IV Reunión de Matemáticos Mexicanos en el Mundo" organized by the CMO-BIRS and Sociedad Matemática Mexicana was one of my best academic experiences as a student. This workshop was interesting, well organized and allowed me to talk with people that are currently studying or working outside of Mexico. [...] I'm sure the contacts and information I gained in this workshop will have a positive impact in my academic future. As a result of this workshop, one participant showed interest in my work and now we're looking to collaborate.*"

- Luis Alberto Lomelí (Instituto de Matemáticas, Pontificia Universidad Católica de Valparaíso, Chile) states: " *Participating in the 2018 Mexican Mathematicians in the World, allowed me to properly network and explore opportunities in México for someone who has chosen a career in Mathematics. I received the help that I needed and met many interesting people. At some point, I did not know if Number Theory was of interest in México, yet I am surprised that there are strong students and postdocs interested in this and related areas. Being currently based at the Math Institute in Valparaíso, Chile, it is important to stay in touch and promote academic exchange.* "

- Fernando Saldaña García (Centro de Investigación en Matemáticas, Mexico) comments: *The CMO-BIRS workshop "IV Reunión de Matemáticos Mexicanos en el Mundo" was an interesting and well-organized workshop. I definitely enjoyed the*

meeting, but the most important part is that I met new people to collaborate. Moreover, I also discovered a wide range of job/postdoc opportunities at institutions outside Mexico. I am sure that my participation in this workshop would improve my short-term future as a mathematician. "

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