Large deviation for extremes in BRW with regularly varying displacements

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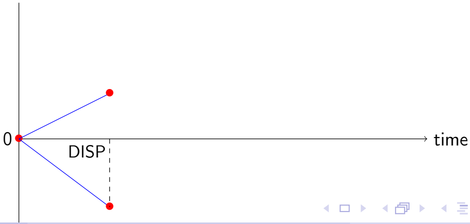

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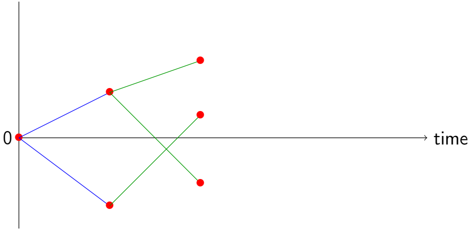

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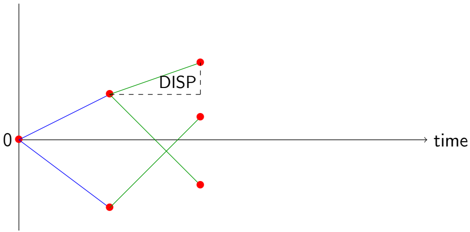

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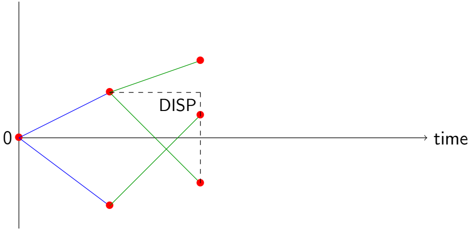

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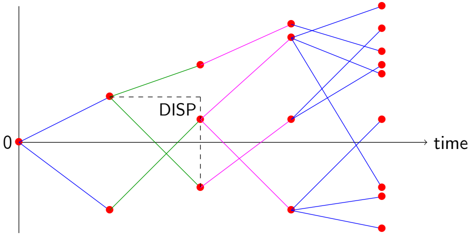
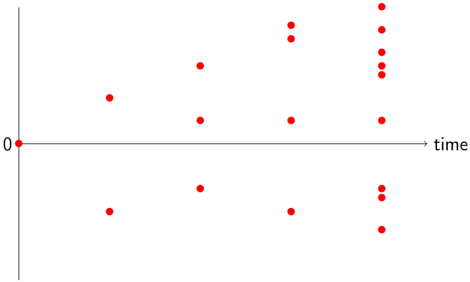
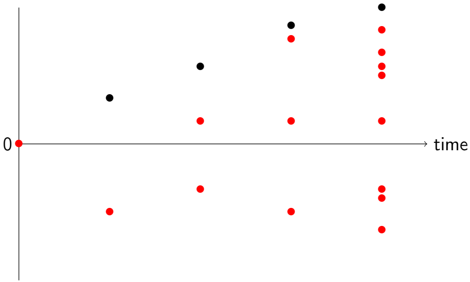


- The collection of positions in the system is called branching random walk (BRW).



- In this talk, we shall focus on the position of the topmost particle in the nth generation.



Why BRW?

- BRW is considered to be very important in the context of probability, statistical physics, algorithms etc. It has connection to Gaussian multplicative chaos, Gaussian free field, random polymers, percolation etc.

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- The displacement of a particle is the lifetime of a bacteria.
- The position of the topmost particle in the nth generation can be interpreted as the last time one can see an nth generation bacteria.

Challenges

- Phase transition in the asymptotic behavior of extremes.

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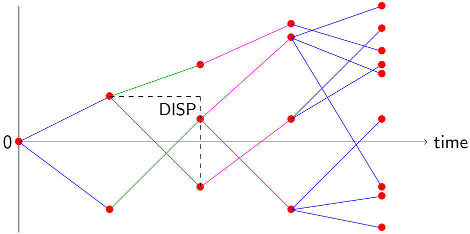
- Reason: Non-trivial dependence structure. (Durrett(1979))



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- We shall assume that the underlying GW process is supercritical and satisfies the Kesten-Stigum condition.
- Z_n denotes the number of particles in the nth generation for every $n \geq 1$.

- $1 < m = \mathbb{E}(Z_1) < \infty$.

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- $(m^{-n} Z_n : n \geq 1)$ is a non-negative martingale sequence and hence $m^{-n} Z_n$ converges to a random variable W almost surely as $n \to \infty$.

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- Kesten-Stigum condition $(\mathbb{E}(Z_1 \log^+ Z_1) < \infty)$ implies that W is positive almost surely due to "no leaf" assumption.

Assumptions on the displacements

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- The displacements are real-valued. For every $x > 0$,

$$\mathbb{P}(|X| > x) = x^{-\alpha} L(x)$$

where L is slowly varying function and satisfies tail-balancing conditions

$$\lim_{x\to\infty} \frac{\mathbb{P}(X > x)}{\mathbb{P}(|X| > x)} = p \quad \text{and} \quad \lim_{x\to\infty} \frac{\mathbb{P}(X < -x)}{\mathbb{P}(|X| > x)} = 1 - p$$

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- Consider a sequence of constants $(b_n : n \geq 1)$ such that $m^n \mathbb{P}(b_n^{-1} X \in \cdot) \xrightarrow{v} \nu_\alpha(\cdot)$ in the space $[-\infty, \infty] \setminus \{0\}$ and

$$\nu_\alpha(dx) = \alpha\Big(p x^{-\alpha-1} \mathbb{1}(x > 0) + (1-p)(-x)^{-\alpha} \mathbb{1}(x < 0)\Big).$$

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- Large deviation is derived for topmost particle in branching Brownian motion (BBM). See Chauvin and Rouault (1988).
- Large deviation for topmost position in different variants of the model BBM: Derrida and Shi(2017).

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Weak convergence of $\mathscr{P}_n$

Theorem (B. Hazra and Roy (2016))

There exists a Cox cluster process $\mathscr{P}$ such that $\mathscr{P}_n \Rightarrow \mathscr{P}$ as $n \to \infty$ in the space $\mathscr{M}$ where

$$\mathscr{P} \stackrel{d}{=} \sum_{l=1}^{\infty} Z_{G_l} \delta_{W^{1/\alpha} j_l}$$

with $(j_l : l \geq 1)$ be the atoms of the PRM(ν_α) on $\mathbb{R}$.

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Question

What is the rate of convergence for $\mathbb{P}(M_n > c_n x)$ *?*

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- Joint distribution of the first k largest positions and gap statistics.
- Consider the sequence of point processes

$$N_n = \sum_{|\mathbf{v}|=n} \delta_{c_n^{-1} S(\mathbf{v})}$$

Question

How does N_n behave asymptotically?

- Hult and Samorodnitsky (2010). Large deviation of extremal processes.

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- Consider $A \subset \mathscr{M}$ such that $\emptyset \notin \bar{A}$. Then it is clear that $\mathbb{P}(N_n \in A) \to 0$.

Question

Does there exist $(r_n : n \geq 1)$ *and a non-trivial measure* λ *on* $\mathscr{M}$ *such that* $r_n \mathbb{P}(N_n \in A)$ *converges to* $\lambda(A)$ *for every nice measurable set* $A \subset \mathscr{M}$*?*

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- We can use M_0 convergence with $\mathbb{S} = \mathscr{M}$ and $s_0 = \emptyset$.

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Consequence: $r_n \mathbb{P}(M_n > c_n x)$ converges to some non-null function f of x. The function f can also be identified.

Literature on large deviation for extremes

- Large deviation results for maxima in BRW with light-tailed displacement (exponentially decaying tail) have been derived by Gantert and Höfelsauer (2018).

- Large deviation for extremal process Hult and Samorodnitsky (2010) and Fasen and Roy (2016). (Regularly varying case).

Main result

Theorem (B. 2018(arXiv:1802.05938v1))

There exists r_n such that for every "nice set" $A \subset \mathscr{M}$,

$$r_n \mathbb{P}(N_n \in A) \xrightarrow{M_0} \lambda(A)$$

where

$$\lambda(A) = \sum_{l=1}^{\infty} m^{-l} \mathbb{E}\Big[\nu_\alpha(x \in \mathbb{R} : Z_l \delta_x \in A)\Big].$$

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- W (martingale limit) does not appear in the limit measure ν.

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Corollary

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Recall that M_n denotes the position of the topmost particle in the nth generation. Then

$$\lim_{n\to\infty} r_n \mathbb{P}(M_n > c_n x) = p \frac{1}{m-1} x^{-\alpha} \qquad \text{for all } x > 0.$$

Proof of consequence: large deviation for maxima

Fix $x > 0$.

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- This can be done for the joint distribution of topmost and bottommost position, first k-order statistics.

Proof strategy: Principle of single large disp.

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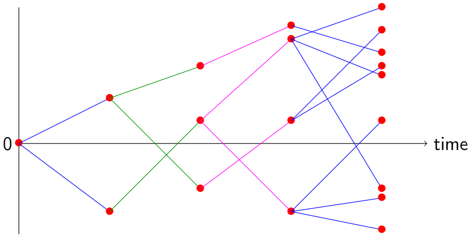
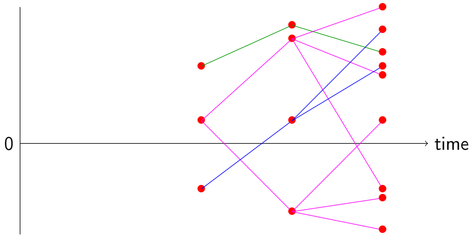
- Step 1 - One large displacement. It is enough to study another point process of the displacements upto nth generation due to at most one large jump in every path.



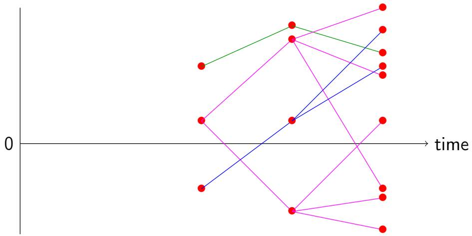
Proof strategy: contd.......

- Step 2 - Cutting the tree (locate the large displacement). Cut the tree at the $(n-K)$th generation and forget whatever happened in the first $(n-K)$ generations. With high probability, one large displacement is contained in the last K generations.



Proof strategy: continued ......

- Advantages of cutting: Get Z_{n-K} independent copies of the independently and identically point processes.
- Each of the subtrees have equal probability to contain the large jump.



Proof strategy: contd.......

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Compute the contribution of the large jump at the Kth generation of the subtrees.

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- Step 3 - Pruning

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- Step 3 - Pruning
- Step 4 - Regularization

Weakening assumptions

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Large deviation for $\mathbb{P}(N_n \in A|$ survival of tree).

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The limit measure λ changes.

Thank you